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Program Book

of

The Seventh Biennial International Group Theory Conference 2023

(7BIGTC 2023)

August 7-11

North-West University

Potchefstroom, South Africa

Preface

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The Biennial International Group Theory Conference (BIGTC) is a series of the conference that is organized every two years and has been hosted by some countries – Malaysia (two times), Turkey, Iran, Indonesia and India. The 7th Biennial International Group Theory Conference 2023 will be held in North-West University, Potchefstroom- South Africa from 7-11 August 2023. The main objectives of this conference are to gather prominent researchers in the field of group theory and be the platform of information on the latest research in this area. The conference also aims to foster interest and motivation in pure mathematics in general and group theory in particular and exchange research ideas among group theorists and postgraduate students. In addition, specialists in the subject of group theory can present their latest research works and encourage interested students to progress and widen their knowledge in group theory. Certainly, this provides great opportunities for the graduate students, junior researchers and specialist in the region, which end up with joint research works internationally.
The First, Second, Third, Fourth, Fifth and Sixth BIGTC have already taken place in Malaysia (UTM Johor Bahru), Turkey (Istanbul), Iran (Mashhad), Malaysia (UTM Kuala Lumpur), Indonesia (ITB, Bandung) and India (VIT, Vellore), respectively.

This book contains a collection of abstracts in the broad subject of group theory. All abstracts, which will be presented in the plenary, invited or parallel talks at the conference, have been collected in this booklet for the convenience of the participants.
We would like to thank all participants and the plenary & invited speakers, who showed interest to attend and give talks in this conference. As the wide range of topics covered by the contributed talks made it necessary to hold different sessions in parallel, so we hope that this collection would in some sense make up the missing talks. We are grateful to the sponsors for their generous financial support. We are grateful to the members of Scientific Committee, the members of organizing committee and all plenary and invited speakers of contributed talks for maintaining the stimulating atmosphere and above all, to all of our participants for making this event an unforgettable one. Finally, we are thankful to all academic and non-academic staff of NWU, who have helped us in organizing this conference.

Prof. Thekiso Seretlo — Organizer of 7BIGTC2023

Prof. Ahmad Erfanian — Chair of 7BIGTC2019 Scientific Committee

Local Organising Committee Members

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- A. Manimaran (India)
- TT. Seretlo, A. Saeidi, and SY. Madanha (South Africa)
- L. Chikamai (Kenya) (Africa)

Plenary Speakers

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- Prof. Bernardo Rodrigue (University of Pretoria, South Africa)
- Prof. Rebecca Waldecker (Institute Martin-Luther-University Halle-Wittenberg, Germany)
- Prof. J. Moori (North-West University ,Mafikeng Campus, South Africa)
- Prof. S. Shpectorov (University of Birmingham, U.K.)

Invited Speakers

- Dr. Abraham Love Prins (Nelson Mandela University, South Africa)
- Dr. Layla Sorkatti (University of Khartoum, Sudan)
- Dr. A. Manimaran (Vellore Institute of Technology, India)
- Prof. Ismail S Guloglu (Dogus¸ University, Turkey)
- Prof. M.R. Darafsheh (University of Tehran, Iran)
- Dr. J. Mclnroy (University of Chester, U.K.)
- Prof. Rajat Kanti Nath (Tezpur University, India)
- Prof A. Iranmanesh (Tarbiat Modares University, Iran)
- Prof Ahmad Erfanian (Ferdowsi University of Mashhad, Iran)
- Prof. Dean Cmkovic (University of Rijeka, Croatia)
- Prof. Cheryl E Praeger (University of Western Australia, Australia)
- Prof. Intan Muchtadi (Institut Teknologi Bandung, Indonesia)
- Prof. N.H. Sarmin (Universiti Teknologi Malaysia, Malaysia)
- Prof D. Kahrobaei (City University of New York, USA)
- Prof. Benard Muthiani Kivunge (Kenyatta University, Kenya)

Program of

The 7th Biennial International Group Theory Conference (7BIGTC 2023)

7 – 11 August, 2023

North-West University (NWU), Potchefstroom, South Africa

MONDAY, 7 August 2023

Time	Program	Venue
8.00 – 9.00	Registration	Building C1, Room 135
9.00 – 09:45	Opening and Formal Speeches	Building C1, Room 135

Time	Title	Plenary Speaker	Venue
09:45 – 10:35	Binary self-dual codes with a prescribed finite imprimitive permutation group	Prof. Bernardo Rodrigue (University of Pretoria, South Africa)	Building C1, Room 135

Time	Program	Venue
10:35 – 10:50	Tea / Coffee Break	Building C1,common areas

Time	Title	Invited Speaker	Venue
10:50 –11:25	On the classes and Fischer matrices of an extension group and its underlying factor group	Dr. Abraham Love Prins (Nelson Mandela University, South Africa)	Building C1, Room 135
11:25–12:00	Symplectic Alternating Algebras	Dr. Layla Sorkatti (University of Khartoum, Sudan)	Building C1, Room 135
12:00-12:25	A Study on The Complements of The Elements of Z-Soft Covering Based Rough Lattice and Its Application	Dr. A. Manimaran (Vellore Institute of Technology, India)	Building C1, Room 135

Time	Program	Venue
12.30 – 14.00	Lunch	Building C1, Dining Room

Parallel Session I

Time	Title	Speaker	Venue
14.00 – 14.20	The ($p; q; r$)-generations and conjugacy classes ranks of group G	Dr. MJ. Motalane (University of Limpopo,South Africa)	Building C1, Room 135
	On some codes and designs from the simple group U3(5)	Dr. Lucy Chikamai (Kibabii University, Kezina)	Building C1, Room 134
14.20 – 14.40	On the ($p;q;r$)-generations of the group $G2(3)$	Mr. MG Sehoana (University of Limpopo,South Africa)	Building C1, Room 135
	A note on the big lattices of classes of R-modules closed under submodules, quotients, and coproducts.	Dr. Philani Majozi (University of Zululand, South Africa)	Building C1, Room 134
14.40 – 15.00	On a Group of the Form $2^{11}:M_{24}$	Mr. Dennis Siwila Chikopela (The Copperbelt University, Zambia)	Building C1, Room 135
	Error correcting codes from 2-representation of unitary group U(3,3)	Mr. Tapiwanashe Gift Nyikadzino (University of Limpopo, South Africa)	Building C1, Room 134

Time	Program	Venue
15.00 – 15.15	Tea / Coffee Break	Building C1,common areas

Time	Program	Venue
15.15 – 17.15	Mathematical Structures and Modelling session	Building C1, Room 135

Time	Program	Venue
18.00 – 20.00 ,	Dinner	Dampad Dining Hall

TUESDAY, 8 August 2023

Time	Title	Plenary Speaker	Venue
8:00 – 8:50	112 years for the proof of one theorem	Prof. Rebecca Waldecker (Institute Martin-Luther-University Halle-Wittenberg, Germany)	Building C1, Room 135

Time	Title	Invited Speaker	Venue
8:50 –9:25	Fixed point free Action of Cyclic Groups	Prof. Ismail S Guloglu (Dogus, University, Turkey)	Building C1, Room 135

Time	Program	Venue
9:25 – 9:40	Tea / Coffee Break	Building C1,common areas

Time	Title	Invited Speaker	Venue
9:40 –10:15	Frobenius Kernel	Prof. M.R. Darafsheh (University of Tehran, Iran)	Building C1, Room 135
10:15 –10:50		Dr. J. Mclnroy (University of Chester, U.K.)	Building C1, Room 135

Time	Program	Venue
10:50- 11:05	Tea / Coffee Break	Building C1,common areas

Time	Title	Invited Speaker	Venue
11.05 –11.40	Graphs and probabilities defined on finite groups	Prof. Rajat Kanti Nath (Tezpur University, India)	Building C1, Room 135

Parallel Session

Time	Title	Speaker	Venue
11:50 – 12.10	Designs invariant under the simple groups $PSp4(q)$	Mr. Clarence Mokalapa (University of Limpopo, South Africa)	Building C1, Room 135
	Chinese Remainder Theorem for Hyper near rings	Prof. Kedukodi Babushri Srinivas (Manipal Institute of Technology, India)	Building C1, Room 134
12.10 – 12.30	On superfluous ideals of N-groups and related graph	Dr. Kuncham Syam Prasad (Manipal Institute of Technology, India)	Building C1, Room 135
			Building C1, Room 134

Time	Program	Venue
12.30 – 14.00	Lunch	Building C1, Dining Room

Time	Program	Venue
14.00 – 15.00	GAP Workshop	

Parallel Session II (online)

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Time	Title	Speaker	Venue
15.00 – 15.20	On the average order of a finite group	Mr. Mihai-Silviu Lazorec (Alexandru Ioan Cuza University, Romania)	Building C1, Room 135
	Hulls of Negacyclic Codes Over F*q* +*v*F*q*	Dr. Sarra Talbi (Sol Plaatje University, South Africa)	Building C1, Room 134
15.20 – 15.40	On the cyclic conjugacy class graph of groups	Dr. Abbas Mohammadian (Ferdowsi University of Mashhad, Iran)	Building C1, Room 135
	Groups having 12-cyclic subgroups	Ms. Khyati Sharma (Shiv Nadar Institution of Eminence, India)	Building C1, Room 134

Time	Program	Venue
15.40 – 15.55	Tea / Coffee Break	Building C1, common areas

Parallel Session III (online)

Time	Title	Speaker	Venue
15.55 – 16.15	Strongly Monolithic Characters of Finite Groups	Mrs. Sultan Bozkurt Gungor (Gebze Technical University, Turky)	Building C1, Room 135
	Generalized Z-fuzzy soft *b*-covering based rough matrices and its application	S. Pavithra (Vellore Institute of Technology, India)	Building C1, Room 134
16.15 – 16.35	On the Parallel and Descriptive Complexities of Group Isomorphism via Weisfeiler–Leman	Dr. Michael Levet (University of Colorado, USA)	Building C1, Room 135
	A classification of rational groups by using character degree graphs	Mrs. Gamze Akar (Istinye University, Turkey)	Building C1, Room 134

16.35 – 16.55'	A Note on Autoconjugate Graphs of Finite Groups	Dr. Masoumeh Ganjali (Ferdowsi University of Mashhad, Iran)	Building C1, Room 135
	Covering alternating groups by powers of cycle classes in symmetric groups	Dr. Sumit Chandra Mishra (IISER Mohali Mohali, Punjab, India)	Building C1, Room 134
16.55 – 17.15	Loops as semidierect product	Mr. Ramjash Gurjar (Central University of Rajasthan Bandar, India)	Building C1, Room 135
	Lattice matrices and linear transformation of lattice vector spaces	Ms P. Pallavi (Manipal Institute of Technology, India)	Building C1, Room 134

Time	Program	Venue
18.00 – 21.00	Social event and Dinner	NWU Botanical Gardens

WEDNESDAY, 9 August 2023

Time	Title	Plenary Speaker	Venue
8.00 – 8.50	Designs and Codes from Finite Groups	Prof. J. Moori (North-West University ,Mafikeng Campus, South Africa)	Building G1, Room 112

Time	Title	Invited Speaker	Venue
8.50 –9.25	On monomial Isaacs π-partial characters of π-separable groups	Prof A. Iranmanesh (Tarbiat Modares University, Iran)	Building G1, Room 112

Time	Program	Venue
9:25- 9:40	Tea / Coffee Break	Building G1, foyer

Time	Title	Invited Speaker	Venue
9:40 –10:15	A graph associated to central automorphisms in a finite group	Prof Ahmad Erfanian (Ferdowsi University of Mashhad, Iran)	Building G1, Room 112
10.15 –10.50	Transitive q-ary designs and q-ary graphs	Prof. Dean Crnkovic (University of Rijeka, Croatia)	Building G1, Room 112

Time	Program	Venue
10:50 – 11.05	Tea / Coffee Break	Building G1, foyer

Time	Title	Invited Speaker	Venue
11.05 –11.40	Group theoretic constructions of normal covers of the complete bipartite graphs K2n,2n	Prof. Cheryl E Praeger (University of Western Australia, Australia)	Building G1, Room 112

Parallel Session Iv

Time	Title	Speaker	Venue
11.50 – 12.10	Normal supercharacter theories of the dicyclic groups	Dr. Hadiseh Saydi (North-West University, South Africa)	Building G1, Room 112
	$Z_2Z_2[u]Z_2[u]$-additive code	Dr. Vadiraja Bhatta (Manipal Academy of Higher Education, Manipal, Karnataka, India)	Building G1, Room 110
12.10 – 12.30	Constructing some designs invariant under alternating groups	Mr. Jan Kekana (University of Limpopo, South Africa)	Building G1, Room 112
	On N-groups with essential ideals and superfluous ideals	Dr. Harikrishnan Panackal (Manipal Academy of Higher Education, Manipal, Karnataka, India)	Building G1, Room 110

Time	Program	Venue
12.30 – 14.00	Lunch	Takeaway Parcels at the Amphi-Theatre

Time	Program	Venue
14.00 – 15.00	GAP Workshop	Building G1, Room 112

Parallel Session V (online)

Time	Title	Speaker	Venue
15.00 – 15.20	Finite groups with many complemented subgroups	Prof. Izabela Agata Malinowska (University of Białystok, Poland)	Building G1, Room 112
	A kind of g-non-commuting graph	Dr.Mahboube Nasiri (Ferdowsi University of Mashhad, Iran)	Building G1, Room 110
15.20 – 15.40	Non-commuting Graph of AC-groups Related to an Automorphism	Mrs. Zeynab Ilbeygi (Ferdowsi University of Mashhad, Iran)	Building G1, Room 112
	Central Automorphisms of Zappa-Szep Products	Mr. Ratan Lal (Central University of Rajasthan, India)	Building G1, Room 110

Time	Program	Venue
15.40 – 15.55	Tea / Coffee Break	Building G1, foyer

Parallel Session VI (online)

Time	Title	Invited Speaker	Venue
15.55 – 16.15	Undeniable Signature Scheme Based on DLFP Over Semiring	S. Sethupathi (Vellore Institute of Technology, India)	Building G1, Room 112
	On the breath of Lie superalgebras	Dr. Afsaneh Shamsaki (University of Damghan, Iran)	Building G1, Room 110
16.15 – 16.35	On the Sylow subgroups of amalgamated free products of pro-Q groups	Dr. Gilbert Mantika (University of Maroua, Kongola,Cameroon)	Building G1, Room 112
	Application of Hypergroup and *Hv*-group on Chemical Urea Formation Reactions	Mr. Fakhry Asad Agusfrianto (Universitas Negeri Jakarta, Indonesia)	Building G1, Room 110
16.35 – 16.55	Constructing Fischer-Clifford Matrices of a maximal subgroup $(2 \times 21{+}8 +){:}(U4(2){:}2)$ of $Fi22$ from its quotient group	Mr. Langson Kapata (Nelson Mandela University, South Africa)	Building G1, Room 112
	Generalised essential ideal graph of an *N*-group	Mrs. Rajani Salvankar (Manipal Institute of Technology, India)	Building G1, Room 110
16.55 – 17.15	On the Planar Property of Ideal-Based Weakly Zero-Divisor Graph of a Commutative Ring	Mr. Asad Ghafoor (University Sultan Zainal Abidin, Malaysia)	Building G1, Room 112
	A Public Key Cryptosystem and a Key Exchange Protocol Based on Invertible Matrices over Semiring	R. Yuvasri (Vellore Institute of Technology, India)	Building G1, Room 110

Time	Program	Venue
17.15 – 18.30	BIGTC Committee Meeting	Building G1, Room 110

Time	Program	Venue
18.00 – 20.00	Dinner	Dampad Dining Hall

THURSEDAY, 10 August 2023

Time	Program	Venue
8.00 – 10.00	GAP workshop	Building C1, Room 135

Time	Program	Venue
10.00 – 10.15	Tea / Coffee Break	Building C1, common areas

Time	Program	Venue
11.00 – 15.00	Accommodations & Lekwena Wildlife Estate	

Time	Program	Venue
17.00 – 18.00	Travel to Gala Dinner	From accommodations to Crista Galli Venue

Time	Program	Venue
18.00 – 21.00	Gala Dinner	Crista Galli Venue

Time	Program	Venue
21.00 – 22.00	Travel to accommodations	From Crista Galli Venue to accommodations

FRIDAY, 11 August 2023

Time	Title	Plenary Speaker	Venue
8:00 – 8:50	Computing automorphism groups of axial algebras	Prof. S. Shpectorov (University of Birmingham, U.K.)	Building C1, Room 135

Time	Title	Invited Speaker	Venue
8:50 –9:25	Probability associated with the verbal subgroup of a finite group	Prof. Intan Muchtadi (Institut Teknologi Bandung, Indonesia)	Building C1, Room 135

Time	Program	Venue
9:25 – 9.40	Tea / Coffee Break	Building C1, common areas

Time	Title	Invited Speaker	Venue
9.40 –10.15	Sheaves of endomorphisms of locally finitely presented modules	Prof. P. Ntumba	Building C1, Room 135
10.15 –10.50	Some Results on Topological Indices of Graphs Associated to Groups and Rings	Prof. N.H. Sarmin (Universiti Teknologi Malaysia, Malaysia)	Building C1, Room 135

Time	Program	Venue
10:50 – 11.05	Tea / Coffee Break	Building C1, common areas

Time	Title	Invited Speaker	Venue
11:05 –11.40	Post-quantum Blockchains using hash functions using higher dimensional special linear groups over finite fields as platforms	Prof D. Kahrobaei (City University of New York, USA)	Building C1, Room 135
11:40 –12.15		Prof. Benard Muthiani Kivunge (Kenyatta University, Kenya)	Building C1, Room 135

Time	Program	Venue
12.30 – 14.00	Lunch	Building C1, Dinning Room

Time	Program	Venue
14.00 – 15.00	Closing	Building C1, Room 135

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Extended Abstracts

of

Plenary Speakers

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Binary self-dual codes with a prescribed finite imprimitive permutation group

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(This relates to joint work with Wolfgang Knapp, Mathematisches Institut, Universität Tübingen, Germany)

Abstract

Given a representation of group elements of a group G by permutations one can work modulo p, p a prime, and obtain a representation of G on a vector space V over GF(p). The invariant submodules (i.e., the subspaces of V taken into themselves by every group element) are then all the p-ary codes C for which G is a subgroup of the automorphism group of C.
One of the questions of current interest in coding theory is the following: given a finite non-solvable permutation group G acting transitively on a set Ω, under what conditions on G are self-dual codes invariant under G existent or nonexistent? In the talk, this problem is investigated under the hypothesis that the group G is a finite imprimitive permutation group.
This talk will give an introduction to this fascinating interplay by focusing on examples, chosen mostly for the illustration of a tool to construct linear code using representation theoretical arguments. In particular, we will determine all binary codes of length 552 which admit the sporadic simple group Co_3 as an imprimitive transitive permutation group. Our aim will be to illustrate the results by using representation theoretical arguments and to discuss the combinatorial properties of the codes as well as their relation to some special properties of the Leech lattice

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group Co_3. For all codes (with two exceptions) we obtain the weight enumerators and in many interesting cases the classification of codewords under the action of the group Co_3. The two exceptions refer to binary self-dual codes, each of which with minimum weight 12.

Keywords:
automorphism group, permutation group, Conway simple groups, representation theory, module, dual module, permutation module.

AMS Mathematics Subject Classification 2020: 05B05 05B20 05B25 05C25 20B25 20B40 20C05 20D08 94B05 94B25

References

[1] W. Knapp and B. G. Rodrigues. On the binary codes of length 552 which admit the simple group Co_3 as a transitive permutation group. Commun. in Algebra. **51** (4) 2023, 1451 - 1461.

112 years for the proof of one theorem

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(Joint work with Volker Remmert)

Abstract

We discuss the Classification of the Finite Simple Groups from several points of view and ask many questions, often from a historical perspective, that still wait to be investigated.

Keywords:
Finite groups; simple groups; classification of finite simple groups, history of group theory.

AMS Mathematics Subject Classification 2020:
20D05; 20-03.

1 Introduction

The theorem that I want to discuss here is the Classification Theorem for the Finite Simple Groups (in short: CFSG). It is difficult to say how long its proof took exactly, in particular because working on its proof meant shaping the theorem itself and its precise statement too, in parallel. Therefore, I decided to start counting at the year 1892, when it was first stated that such a classification would be desirable (see [5]), and I stopped counting at the year 2004, when the last major gap was closed with the classification of the so-called quasi-thin groups (see [1]). Here are some obvious questions:

- What did group theory look like before this serious and focused classification effort?

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- Why is it desirable to know all the finite simple groups?
- Why did it take so long to prove the classification theorem?
- What was the impact of the CFSG?

The following questions might be less obvious:

- When did the CFSG change from a very ambitious, vague idea to a conjecture and then to a theorem that was widely believed, even when the proof was still not complete?
- How did the organisation of this major mathematical project even work?
- Where did the funding for many of the related PhD projects, for the conferences and for the special programmes come from?
- How did this work change the attitude towards using computers for mathematical work, and the perception of what a proof is?
- When exactly, how and why did the communal sense shift from the excitement around the discovery of finding new groups to a belief that everything had been found and then to the belief that the proof was (at least, fundamentally) complete and correct?

2 Background and the statement of the theorem

Whenever groups are mentioned now, in the remainder of this note, then they are supposed to be finite. It is indeed one of the challenging and fascinating aspects of group theoretic research that the field is so specialised now and so widely spread, with methods and relevant applications varying enormously between infinite groups and finite groups, abelian groups and non-abelian groups, soluble groups and non-soluble groups. In hindsight, one of the first properties that became interesting might have been solubility, in connection with elementary algebraic formulae for roots of polynomials. It was work by Abel and Galois that clarified why such elementary algebaric formulae do not exist, at least not in general, for polynomials of degree 5 and higher. The systematic investigation of symmetry was the foundation of what we no call finite group theory, and a connection was found between a property of the Galois Group related to a polynomial (the property that we now call solubility) and the existence of algebraic formulae for the roots of such a polynomial. And once there were examples of non-soluble groups, it was a natural question to ask

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how abundant they are. The finite simple groups, i.e. the groups with exactly two normal subgroups, are the ones that contribute to the abundance of non-soluble groups in the sense that every simple soluble group is abelian (even cyclic of prime order), leading to a potentially rich theory of finite groups beyond solubility once it is better understood what is so special about the simple groups. According to [5], the classification question was first asked in 1892, by Otto Hölder: :It would be of the greatest interest if it were possible to give an overview of the entire collection of finite simple groups."

When it comes to the relevance of this problem, then my favourite comparison is inspired by chemistry. It might not be enough to know all the atoms (or the elements), but without them there is no way to understand how molecules are built and how we can construct new, previously unknown molecules (and hence materials, for example).

Here it is, the list of the "atoms" of group theory:

Theorem 2.1. *Every finite simple group is isomorphic to one of the following:*

1. *a cyclic group of prime order,*
2. *an alternating group of degree at least 5,*
3. *a finite simple group of Lie type or*
4. *one of 26 sporadic simple groups.*

More details and a strategy for a proof can, for example, be found in [3]. It is striking how short and elegant the statement of the CFSG looks, right? The complexity is hidden, partly in the description of the groups that are mentioned, once we make it explicit and exact. But even then, the statement does not give away how long, how difficult and how intertwined the proof is and how much theory was developed over many decades in order to come closer to it. And with applications in mind, it is not enough to have a list of possibilities. We need detailed information about the structure of the groups, their subgroups, possible actions and their representation theory.

3 Applications, impact and final comments

Out of a wide range of applications, let us focus on one area that has always been very close to abstract finite group theory and where the impact of the CFSG is tangible: representation theory. In an extensive and very approachable overview (see [4]), Malle discusses several local-global conjectures and the progress that has been made, and he points out, in many places, the exact connection to the CFSG. For a number of these famous open conjectures, there was evidence that it might be possible to reduce the question to finite simple groups. There, it might be possible to prove the conjecture, which turned out to be

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true at least in some cases (e.g. the McKay Conjecture for the prime 2, and one direction of Brauers Height Zero Conjecture). The close interplay between abstract local group theory and representation theory, both ordinary and modular, is visible in many places in the proof of the CFSG itself, too, as is the variety of contexts where groups appear and how we can view and investigate them. Group actions on geometric objects or graphs have not only been at the origin of abstract group theory, but they have been a rich source of inspiration for decades, leading to extensive theory (often relevant to the CFSG) and to many new research questions.

We have barely touched the surface, and already the impact of the CFSG becomes visible in several areas of mathematics, and theorem after theorem is proved based on this powerful foundation. We learn more and more details about the groups from the CFSG list themselves, we keep rephrasing and reproving, and over the years we become more and more confident that the result is really true. Of course we could ask why there should be reason for doubt, at all, but maybe this is obvious: After all, there was still a substantial gap after the first announcement, and who is really capable of understanding all the details of a proof that spans over generations of mathematicians and many thousand pages of published work? What does this tell us about what we mean when we say "proof"? This question becomes relevant even earlier, namely when the famous "Odd Order Theorem" appeared (see [2]), proving that groups of odd order are soluble. The result is a milestone towards a feasible strategy for classifying all finite simple groups, and its proof is very long and intricate. The use of computers, the complexity and technical depth of a number of background results and the overarching strategy for the proof of the CFSG itself, with all its sub-cases and technical details, challenges our perception of "proof". This means that, along with a large number of historical questions around the CFSG, we also have philosophical questions to consider – all while celebrating this astonishing achievement of modern mathematics.

References

[1] M. Aschbacher and S. D. Smith, *The Classification of Quasithin Groups (I, II), Mathematical Surveys and Monographs*, **111, 112**, AMS, Providence, RI, (2004).

[2] W. Feit and J,. G. Thompson, *Solvability of groups of odd order, Pacific Journal of Mathematics*, **13** (1963), 775–1029.

[3] D. Gorenstein, R. Lyons and R. Solomon, *The classification of the finite simple groups. Mathematical Surveys and Monographs*, **40.1.**, AMS, Providence, RI, (1994).

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[4] G. Malle, *Local-global conjectures in the representation theory of finite groups, In: Representation theory - current trends and perspectives. Eur. Math. Soc.*, Zrich (2017), 519-539.

[5] R. Solomon, *A Brief History of the Classification of the Finite Simple Groups. In Bulletin (New Series) of the American Mathematical Society*, **38**, No. 3 (2001), 316–352.

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Designs and Codes from Finite Groups

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Abstract

We will discuss three methods for constructing designs and codes from finite groups, mostly simple finite groups. This is a survey on Methods 1 and 2 (see [1]) and on the new method (Method 3, which was introduced by the author in [2]) for constructing designs and codes from fixed points of elements of finite transitive groups. The background material and results required from finite groups, permutation groups and representation theory will be presented in details in the main presentation.

Keywords:
Codes; Designs; Finite Groups; Transitive; Fixed Points.

AMS Mathematics Subject Classification 2020:
20D05-05B0520

References

[1] J. Moori, *Finite groups, designs and codes*, Information Security, Coding Theory and Related Combinatorics, Nato Science for Peace and Security Series D: Information and Communication Security, **29** (2011), 202–230, IOS Press (ISSN 1874-6268).

[2] J. Moori, *Designs and codes from fixed points of finite groups*, Communications in Algebra, **49** (2021), 706–720.

[1]**Online Speaker**

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Extended Abstracts

of

Invited Speakers

On the classes and Fischer matrices of an extension group and its underlying factor group

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Abstract

Let K be a normal subgroup of a finite group extension $\overline{G} = P.G$ such that K is characteristic in P. It is a well-known fact that the ordinary irreducible characters $\mathrm{Irr}(\overline{F})$ of the factor group $\overline{F} = \frac{\overline{G}}{K}$ can be lifted to $\overline{G}$, where the set $\mathrm{Irr}(\overline{F})$ is identified with $\chi_i \in \mathrm{Irr}(\overline{G})$ such that $K \leq \ker(\chi_i)$. In this talk, we will use an analogous *lifting process* to construct a so-called Fischer matrix $M(g)$ of $\overline{G}$ from the corresponding Fischer matrix $\widehat{M(g)}$ of the factor group $\overline{F}$, where g is class representative in G. Therefore, obtaining the matrices $M(g)$ for each class representative g of G, the ordinary character table of $\overline{G}$ can be assembled using the Fischer matrices technique. **Keywords:**
Fischer matrices; extension group; lifting ; character; Factor group.

AMS Mathematics Subject Classification 2020:
20C15-20C40

1 Introduction

Let $\overline{G} = P.G$ be a finite extension of a normal p-subgroup P of $\overline{G}$ by a group G. If K is a non-trivial characteristic subgroup of P then $K \triangleleft \overline{G}$. Hence we obtain the structures $\overline{G} = K.\overline{F}$ and $\overline{F} = \frac{\overline{G}}{K} \cong P_1.G$, where $P_1 \cong \frac{P}{K}$. The commutative diagram, depicted as Figure 1, is associated with the structures $\overline{G}$, $\overline{F}$ and G, where η_1, η_2 and $\eta = \eta_2 \circ \eta_1$ are the natural homomorphisms from $\overline{G}$ onto $\overline{F}$, $\overline{F}$ onto G and $\overline{G}$ onto G, respectively.

$$
\begin{tikzcd}
\overline{G} \arrow[r, "\eta_1"] \arrow[dr, "\eta=\eta_2\circ\eta_1"'] & \overline{F} \arrow[d, "\eta_2"] \\
& G
\end{tikzcd}
$$

Figure 1

1.1 Relationship between the conjugacy classes of $\overline{G}$, $\overline{F}$ and G

Let's consider the group $\overline{F}$, then G is identified with $\frac{\overline{F}}{P_1}$ under the map η_2. Moreover, under the map η_2, the pre-image of a conjugacy class $[P_1\overline{q}]$ in $\frac{\overline{F}}{P_1}$ is a union $\bigcup_{i=1}^{\widehat{c(g)}}[\overline{q_i}]$ of say $\widehat{c(g)}$ conjugacy classes $[\overline{q_i}]$ in $\overline{F}$. Note that each coset $P_1\overline{q}$ can be identified with a $g \in G$ such that $\overline{q}$ is a lifting for g. Therefore, corresponding to a representative $P_1\overline{q} \in [P_1\overline{q}]$ (or a class representative $g \in G$) there is a set $\widehat{X(g)} = \{\overline{q_1} = \overline{q}, \overline{q_2}, \ldots, \overline{q_{\widehat{c(g)}}}\}$ of representatives of conjugacy classes $[\overline{q_i}]$ of $\overline{F}$. Similarly, a pre-image of a class $[\overline{q_i}]$ of $\overline{F}$, with $\overline{q_i} \in \widehat{X(g)}$, under the map η_1 will be a union $\bigcup_{j=1}^{c(\overline{q_i})}[\overline{g_{ij}}]$ of $c(\overline{q_i})$ classes $[\overline{g_{ij}}]$ of $\overline{G}$. Note that $\overline{q_i} \in \widehat{X(g)}$ is identified with a coset $K\overline{g_i} \in \frac{\overline{G}}{K} \cong \overline{F}$ where $\overline{g_i}$ is a lifting for $\overline{q_i}$ in $\overline{G}$. Hence a set $X(\overline{q_i}) = \{\overline{g_{i_1}} = \overline{g_i}, \overline{g_{i_2}}, \ldots, \overline{g_{i_{c(\overline{q_i})}}}\}$ of representatives of conjugacy classes $[\overline{g_{i_j}}]$ is obtained from the coset $K\overline{g_i}$ (or equivalently a class representative $\overline{q_i} \in \overline{F}$). Since $\eta = \eta_2 \circ \eta_1$ (see Figure 1), it follows that a pre-image for $g \in G$ under the map η is a set

$\overline{X(g)} = \bigcup_{i=1}^{\widehat{c(g)}} X(\overline{q_i}) = X(\overline{q_1}) \bigcup X(\overline{q_2}) \bigcup \ldots \bigcup X(\overline{q_{\widehat{c(g)}}})$

$= \{\overline{g_{1_1}} = \overline{g_1}, \overline{g_{1_2}}, \ldots, \overline{g_{1_{c(\overline{q_1})}}}, \overline{g_{2_1}} = \overline{g_2}, \overline{g_{2_2}}, \ldots, \overline{g_{2_{c(\overline{q_2})}}}, \ldots, \overline{g_{\widehat{c(g)}_1}} = \overline{g_{\widehat{c(g)}}}, \overline{g_{\widehat{c(g)}_2}}, \ldots, \overline{g_{\widehat{c(g)}_{c(\overline{q_{\widehat{c(g)}}})}}}\}$

1.2 Fischer matrices of $\overline{F}$

Suppose that $\overline{F}$ has t orbits on $\mathrm{Irr}(P_1)$ with corresponding inertia groups $\overline{H}_s = P_1.H_s = \left\{\overline{q} \in \overline{F} | \theta_s^{\overline{q}} = \theta_s\right\}$, $s = 1, 2, \ldots t$, where $\theta_s \in \mathrm{Irr}(P_1)$ is an orbit representative and $\frac{\overline{H}_s}{P_1} \cong H_s \leq G$. Now each θ_s extends to a projective character $\psi_s \in \mathrm{IrrProj}(\overline{H}_s, \overline{\alpha}_s)$ with factor set $\overline{\alpha}_s$, i.e. $\psi_s \downarrow_{P_1} = \theta_s$. Furthermore, an ordinary irreducible character $\chi = (\psi_s\overline{\beta})^{\overline{F}}$ of $\overline{F}$ is obtained by induction of $\psi_i\overline{\beta} \in \mathrm{Irr}(\overline{H_s})$ to $\overline{F}$. Since $\overline{\alpha}_s$ is constant on cosets of P_1 in $\overline{H}_s$, it can be identified as a factor set α_s of the inertia factor H_s. Moreover, $\overline{\beta}$ is a lift for $\beta \in \mathrm{IrrProj}(H_i, \alpha_i^{-1})$ to $\overline{H_i}$ and the set $\mathrm{IrrProj}(H_i, \alpha_s^{-1})$ can be obtained from the ordinary irreducible characters $\mathrm{Irr}(\overline{H}_s)$ which contain θ_s as an irreducible constituent on their restriction to P_1. By Gallagher [2], we obtain

$$\mathrm{Irr}(\overline{F}) = \bigcup_{s=1}^{t} \{(\psi_s\overline{\beta})^{\overline{F}} \mid \beta \in \mathrm{IrrProj}(H_s, \alpha_s^{-1})\}.$$

Hence the set $\mathrm{Irr}(\overline{F})$ is partitioned into t blocks B_s with each block B_s corresponding to an inertia group $\overline{H_s}$. Observe that $|\mathrm{Irr}(\overline{F})| = |\mathrm{Irr}(H_1)| + |\mathrm{IrrProj}(H_2, \alpha_2^{-1})| + \ldots + |\mathrm{IrrProj}(H_t, \alpha_t^{-1})|$.

We take $\overline{H_1} = \overline{F}$ and $H_1 = G$. We define the set

$$R(g) = \{(s, y_k) \mid 1 \leq i \leq t, H_s \cap [g] \neq \emptyset, 1 \leq k \leq r\},$$

where y_k, $k = 1, 2, ..., r$, are representatives of the α_s^{-1}- regular conjugacy classes $[y_k]$ of H_s that fuse into a class $[g]$ of $H_1 = G$. Let y_{l_k} be representatives of the conjugacy classes of $\overline{H}_s$, where each y_{l_k} has y_k as an image under the homomorphism $\overline{H_s} \longrightarrow H_i$ whose kernel is P_1. Also, we let $\widehat{X(g)} = \{\overline{q_1} = g, \overline{q_2}, \ldots, \overline{q_{\widehat{c(g)}}}\}$ be the set which contains the representatives of the classes of $\overline{F}$ coming from $P_1 g$. Then for $\overline{q_i} \in \widehat{X(g)}$, we have

Lemma 1.1.

$$(\psi_s\overline{\beta})^{\overline{F}}(\overline{q_i}) = \sum_{y_k:(s,y_k)\in R(g)} \left[\sum_{l}' \frac{|C_{\overline{F}}(\overline{q_i})|}{|C_{\overline{H_s}}(y_{l_k})|} \psi_s(y_{l_k}) \right] \beta(y_k)$$

Proof. See [3] □

A Fischer matrix $\widehat{M(g)} = \left(a^i_{(s,y_k)}\right)$ of $\overline{F}$ [1] is then defined as

$$\left(a^i_{(s,y_k)}\right) = \left(\sum_{l}' \frac{|C_{\overline{F}}(\overline{q_i})|}{|C_{\overline{H_s}}(y_{l_k})|} \psi_s(y_{l_k}) \right),$$

with columns indexed by $\widehat{X(g)}$ and rows indexed by $R(g)$ and where $\sum_l'$ is the summation over all l for which y_{l_k} is conjugate to $\overline{q_i}$ in $\overline{F}$.

The Fischer matrix $\widehat{M(g)}$ (see Figure 2) is partitioned row-wise into blocks $\widehat{M_s(g)}$, where each block corresponds to an inertia group $\overline{H_s}$. We write $|C_{\overline{F}}(\overline{q_i})|$, for each $\overline{q_i} \in \widehat{X(g)}$, at the top of the columns of $\widehat{M(g)}$ and at the bottom we write $m_i = [C_g{:}C_{\overline{F}}(\overline{q_i})] = |P_1| \frac{|C_G(g)|}{|C_{\overline{F}}(\overline{q_i})|}$ and $C_g = \{x \in \overline{F} | x(P_1 g) = (P_1 g)x\}$. On the left of each row we write $|C_{H_s}(y_k)|$, where the α_s^{-1}-conjugacy classes $[y_k]$, $k = 1, 2, ..., r$, of an inertia factor H_s fuse into the conjugacy class $[g]$ of G. Since $|\widehat{X(g)}| = |R(g)|$ it follows that $\widehat{M(g)}$ is a square matrix of size $\widehat{c(g)}$. When there is no class fusion of an inertia factor H_s into a class $[g]$, the block $\widehat{M_s(g)}$ is omitted from $\widehat{M(g)}$.

$$
\begin{array}{cc}
 & \begin{array}{cccc} |C_{\overline{F}}(\overline{q_1})| & |C_{\overline{F}}(\overline{q_2})| & \cdots & |C_{\overline{F}}(\overline{q_{\widehat{c(g)}}})| \end{array} \\
\begin{array}{c} |C_G(g)| \\ |C_{H_2}(y_1)| \\ |C_{H_2}(y_2)| \\ \vdots \\ |C_{H_s}(y_1)| \\ |C_{H_s}(y_2)| \\ \vdots \\ |C_{H_t}(y_1)| \\ |C_{H_t}(y_2)| \\ \vdots \end{array} &
\left(\begin{array}{cccc}
1 & 1 & \cdots & 1 \\ \hline
a^1_{(2,y_1)} & a^2_{(2,y_1)} & \cdots & a^{\widehat{c(g)}}_{(2,y_1)} \\
a^1_{(2,y_2)} & a^2_{(2,y_2)} & \cdots & a^{\widehat{c(g)}}_{(2,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\ \hline
a^1_{(s,y_1)} & a^2_{(s,y_1)} & \cdots & a^{\widehat{c(g)}}_{(s,y_1)} \\
a^1_{(s,y_2)} & a^2_{(s,y_2)} & \cdots & a^{\widehat{c(g)}}_{(s,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\ \hline
a^1_{(t,y_1)} & a^2_{(t,y_1)} & \cdots & a^{\widehat{c(g)}}_{(t,y_1)} \\
a^1_{(t,y_2)} & a^2_{(t,y_2)} & \cdots & a^{\widehat{c(g)}}_{(t,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\ \hline
\end{array}\right) \\
 & \begin{array}{cccc} m_1 & m_2 & \cdots & m_{\widehat{c(g)}} \end{array}
\end{array}
$$

Figure 2: The Fischer Matrix $\widehat{M(g)}$

2 Main results

Since $\overline{F}$ is a factor group of $\overline{G}$, the set $\mathrm{Irr}(\overline{F})$ can be lifted to $\overline{G}$. It follows that Theorem 2.1 can be formulated as below.

Theorem 2.1. *For a class representative $g \in G$, a Fischer matrix $\widehat{M(g)}$ of the factor group $\overline{F}$ is embedded in the corresponding Fischer matrix $M(g)$ of $\overline{G}$.*

In a sense, we say that the matrix $M(g)$ is a *lift* for $\widehat{M(g)}$ to $\overline{G}$. In this talk, the construction of a Fischer matrix $M(g)$ of $\overline{G}$ from the corresponding Fischer matrix $\widehat{M(g)}$ of $\overline{F}$ is discussed and together with the character tables (ordinary or projective) of the inertia factors H_s of the action of G on $\mathrm{Irr}(P)$, the ordinary character table of $\overline{G}$ can be constructed via the Fischer matrices technique [1]. In particular, when P is nonabelian and P_1 an elementary p-group which is a G-module over $GF(p)$, then this lifting method is very powerful. Appropriate examples of extension groups with nonabelian kernels will be discussed to illustrate this *lifting method of Fischer matrices*.

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Symplectic Alternating Algebras

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Abstract

In 2008, Traustason discovered SAAs. This was originated from a work on powerful 2-Engel 3-groups, where a certain class that have a richer structure has arisen. This lead to what we call *Symplectic Alternating Algebras*. In this talk, I will present these algebras in their own rights and discuss many beautiful properties that holds for these algebras.

Moreover, in 2012, Tortora, Tota and Traustason were looking at nil-algebras of dimension 8. The study reveals that they were all nilpotent. We then focus on the nilpotent subclass which is a natural subclass to study. I also will report some developed structure theory for nilpotent symplectic alternating algebras.

Keywords: Non-associative algebras; Symplectic, Nilpotent; Alternating; Engel.

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1 Introduction

We give the definition of a symplectic alternating algebra and nilpotent symplectic alternating algebra with some examples and non-examples.

[1] **Online Speaker**

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1.1 What is a Symplectic Alternating Algebra?

A Symplectic Alternating Algebra (SAA) over a field F is a Symplectic vector space L over F with a non-degenerate alternating form $(,)$ equipped with a binary alternating product $\cdot$ such that the symplectic peoperty

$$(u\cdot v,w) = (v\cdot w,u),$$

holds for all $u,v,w \in L$.

Suppose we have any basis $u_1,\dots,u_{2n}$ for L. The structure of L is then determined from

$$(u_iu_j,u_k) = \gamma_{ijk}, \quad 1 \le i < j < k \le 2n.$$

We refer to such data as a *presentation* for L. Alternatively we can describe L as follows, if we take the two isotropic subspaces $Fx_1+\cdots+Fx_n$ and $Fy_1+\cdots+Fy_n$ with respect to a given standard basis, then it is suffices to write down only the products x_ix_j, y_iy_j, $1 \le i < j \le n$. The reason for this is that having determined these products we have determined all the triples (u_iu_j,u_k) where $1 \le i < j < k \le 2n$, since two of those are either some x_i,x_j or some y_i,y_j in which case the triple is determined from x_ix_j or y_iy_j. Since $(x_ix_j,x_k) = (x_jx_k,x_i) = (x_kx_i,x_j)$ and $(y_iy_j,y_k) = (y_jy_k,y_i) = (y_ky_i,y_j)$, this put some more conditions on the products x_ix_j and y_iy_j.

1.2 Examples.

It is clear that the only SAA of dimension 2 is the abelian one. Furthermore, it is easily seen that apart from the abelian one there is only one SAA of dimension 4 that can be described by the following multiplication table. (see [10]).

$$\begin{array}{ll} \mathrm{x}_1x_2 = 0 & \mathrm{x}_1y_2 = -x_1 \\ y_1y_2 = -y_1 & x_2y_1 = 0 \\ x_1y_1 = x_2 & x_2y_2 = 0 \end{array}$$

The presentation is thus $(x_1y_1,y_2) = 1$. In general, there is a close connection between SAA's over the field GF(3) of three elements and a certain class of 2-Engel groups, and in [10] the SAA's over GF(3) of dimension 6 were classified.

We define the lower central series in an analogous way to related structures like associative algebras and Lie algebras. Thus we define the lower central series recursively by

$$L^1 = L \quad \& \quad L^{n+1} = L^n \cdot L,$$

and the upper central series by

$$Z_0(L) = \{0\} \quad \& \quad Z_{n+1}(L) = \{x \in L : x\cdot L \in Z_n(L)\}.$$

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It is readily seen that the terms of the lower and the upper central series are all ideals of L. We however have a special relation that will be used frequently between the upper and the lower central series as follows:

$$Z_n(L) = (L^{n+1})^{\perp}.$$

Some general theory was developed in [2, 3, 4, 5, 6, 7, 8, 10, 11]. In particular a well-known dichotomy property for Lie algebras also holds for SAA's. Thus a SAA is either semi-simple or has a non-trivial abelian ideal. Furthermore, the notion of *rank* can be defined and also there exist a non-trivial SAA L where $\operatorname{Aut} L = \{\mathrm{id}\}$.

1.3 What is a Nilpotent Symplectic Alternating Algebra?

The definition of nilpotent algebras is standard. Thus, A symplectic alternating algebra L is *nilpotent* if there exists an ascending chain of central ideals $I_0, \ldots, I_n$ such that

$$\{0\} = I_0 \leq I_1 \leq \cdots \leq I_n = L$$

and $I_s L \leq I_{s-1}$ for $s = 1, \ldots, n$. The smallest possible n is then called the *nilpotence class* of L. Equivalently we have that L is nilpotent of class $n \geq 0$ if n is the smallest non-negative integer such that $L^{n+1} = \{0\}$ or equivalently $Z_n(L) = L$.

1.4 Examples and non-examples

The non-abelian SAA of dimension 4 is not nilpotent whereas the symplectic algebras of dimension 6 with the presentation $(y_1 y_2, y_3) = 1$ is nilpotent.

2 Main results

In this section we describe one of the main crucial results that provide a great deal about the structure of the nilpotent sub-class.

Lemma 2.1. *Let L be a SAA of dimension $2n$. Any one-dimensional ideal of L is contained in $Z(L)$ and any two-dimensional ideal is abelian and contains a non-trivial element from $Z(L)$.*

Proposition 2.2. *Let L be a SAA of dimension $2n$. No term of the upper central series has co-dimension 1. Equivalently, no term of the lower central series has dimension 1.*

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Theorem 2.3. *Let L be a nilpotent SAA of dimension $2n \geq 2$. There exists an ascending chain of isotropic ideals*

$$\{0\} = I_0 < I_1 < \cdots < I_{n-1} < I_n$$

such that $\dim I_r = r$ for $r = 0, \ldots, n$. Furthermore, for $2n \geq 6$, $I_{n-1}^{\perp}$ is abelian and the ascending chain

$$\{0\} < I_2 < I_3 < \cdots < I_{n-1} < I_{n-1}^{\perp} < I_{n-2}^{\perp} < \cdots < I_2^{\perp} < L$$

is a central chain. In particular L is nilpotent of class at most $2n-3$.

Next we apply the previous theorem to the standard basis $x_1, y_1, x_2, y_2, \ldots, x_n, y_n$ where

$$I_1 = Fx_n,\ I_2 = Fx_n + Fx_{n-1},\ \ldots,\ I_n = Fx_n + \cdots + Fx_1,$$

$$I_{n-1}^{\perp} = I_n + Fy_1,\ I_{n-2}^{\perp} = I_n + Fy_1 + Fy_2,\ \ldots,\ I_0^{\perp} = L = I_n + Fy_1 + \cdots + Fy_n.$$

Now let u, v, w be three of the basis elements. Since I_n is abelian we have that $(uv, w) = 0$ whenever two of these three elements are from $\{x_1, \ldots, x_n\}$. The fact that

$$\{0\} < I_1 < \cdots < I_n$$

is central also implies that $(x_i y_j, y_k) = 0$ if $i \geq k$. It follows that we only need to consider the possible non-zero triples $(x_i y_j, y_k)$, $(y_i y_j, y_k)$ for $1 \leq i < j < k \leq n$. For each triple (i, j, k) with $1 \leq i < j < k \leq n$, let $\alpha(i, j, k)$ and $\beta(i, j, k)$ be some elements in the field F. We refer to the data

$$(x_i y_j, y_k) = \alpha(i, j, k), \quad (y_i y_j, y_k) = \beta(i, j, k), \quad 1 \leq i < j < k \leq n$$

as a *nilpotent presentation*. We have just seen that every nilpotent SAA has a presentation of this type. Conversely, given any nilpotent presentation, let

$$I_r = Fx_n + Fx_{n-1} + \cdots + Fx_{n+1-r}$$

and we get an ascending central chain of isotropic ideals $\{0\} = I_0 < I_1 < \cdots < I_n$ such that $\dim I_j = j$ for $j = 1, \ldots, n$. It follows that we then get a central chain

$$\{0\} = I_0 < I_1 < \cdots < I_n < I_{n-1}^{\perp} < I_{n-2}^{\perp} < \cdots < I_0^{\perp} = L$$

and thus L is nilpotent. Thus every nilpotent presentation describes a nilpotent SAA.

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A Study on The Complements of The Elements of Z-Soft Covering Based Rough Lattice and Its Application

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Abstract

In this paper, we discuss information system $I = (\Omega, B)$ and related Z-soft covering based rough lattices $(T_S, \vee, \wedge)$ where $\vee$ denotes join and $\wedge$ denotes meet. We prove that there exist maximal and minimal elements of a Z-soft covering based rough lattice and define the complement of elements of the set T_S. The proposed concepts are explained through examples.

Keywords:
Soft set ; Rough set; Soft covering based rough set; Lattice; Boolean algebra.

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Fixedpointfree Action of Cyclic Groups

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Abstract

Let A be a finite nilpotent group acting fixed point freely on the finite (solvable) group G by automorphisms. It is conjectured that the nilpotent length of G is bounded above by $\ell(A)$, the number of primes dividing the order of A counted with multiplicities. In the present paper we consider the case A is cyclic and obtain that the nilpotent length of G is at most $2\ell(A)$ if $|G|$ is odd.

Keywords:
nilpotent length; automorphism; fixed point free action .

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1. Introduction

Let G be a finite group and A another finite group acting on G by automorphisms. The set of fixed points of this action is denoted by $C_G(A) = \{g \in G : g^a = g \text{ for all } a \in A\}$ and we say that this action is fixed-point-free if $C_G(A) = 1$. The information that G admits a group with fixed-point-free action on it gives also a lot of information about the structure of G .
The famous theorem of **Thompson** showing that G has to be nilpotent if it admits a fixed-point-free group of automorphisms of prime order is a typical result along these lines. If A is a finite nilpotent group acting fixed-point-freely on the finite group G, then one can regard A as a Carter subgroup of the semidirect product GA with normal complement G. This observation and the above mentioned result led Thompson to conjecture that the nilpotent length of a solvable group H is bounded by a function of $\ell(C)$ which

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denotes the number of (not necessarily distinct) prime divisors of the order of a Carter subgroup C of H. This conjecture is affirmatively answered by **Dade** in his 1969's paper [**3**] which gives an exponential bound for the nilpotent length $h(H)$ in terms of $\ell(C)$. In the same paper Dade formulates the following conjecture :

Let A be a finite nilpotent group acting fixed-point-freely on the finite group G by automorphisms. Then $h(G)$ is bounded above by a linear function of $\ell(A)$.

No linear bound has been found so far even if A is cyclic although much progress has been obtained in establishing the truth of this conjecture under the additional assumption that $(|G|,|A|) = 1$. All the results in this direction up to 1994 are listed in the survey paper [**12**] of **Turull**. We can summarize the best description of the results obtained so far by giving Theorem 2.1 in [**13**] as follows:

Let A be a finite group acting coprimely on the finite solvable group G by automorphisms. Assume that for every subgroup A_0 of A and every A_0-invariant irreducible elementary abelian section S of G there is $v \in S$ with $C_{A_0}(v) = C_{A_0}(S)$. Then

$$h(G) \leq \ell(A) + \ell(C_G(A)).$$

Notice that this yields immediately the bound $\ell(A)$ in case where $C_G(A) = 1$. This is the best bound we can hope since it sis known for any finite group A there is a finite solvable group G with $h(G) = \ell(A)$ on which A acts coprimely and fixed-point-freely.

As an example to the noncoprime case we can give a result of Turull, namely the following.

Let A be a finite abelian group of squarefree exponent acting fixed point freely on the finite solvable group G by automorphisms. Then $h(G) \leq 5\ell(A)$.

On the same lines the authors obtained in [**5**] that $h(G) \leq \ell(A)$ under the assumption that A is finite abelian of squarefree exponent coprime to 6 acting fixed point freely on the group G of odd order.

More recently, **Jabara** handled the case where A is cyclic in [**7**], and obtained the polynomial bound

$$h(G) \leq 7\ell(A)^2.$$

In 1990 [**1**] **Bell** and **Hartley** constructed an elegant example showing that the nilpotentness condition on A cannot be freely dropped in case of a noncoprime action. Namely, they proved the following

Fact. *For any finite nonnilpotent group A and a positive integer k, there exists a finite solvable group G on which A acts fixed point freely and $h(G) = k$.*

In view of this result the conjecture should be restated as follows.

Conjecture. *Let A be a finite nilpotent group acting fixed point freely on the finite solvable group G by automorphisms. Then*

$$h(G) \leq \ell(A).$$

It should be noted that actually the solvability of G is guaranteed if G admits a fixed-point-free group of automorphisms by a result of Rowley [**9**] if the action is coprime and by a result of Belyaev-Hartley [**2**] in the case of nilpotentcy of the acting group.

It is our aim in the present paper to obtain a result in the noncoprime situation which is similar to Theorem 2.1 in [**13**] mentioned above when A is cyclic. Let A act on the group G, and let $\mathbf{c}(G;A)$ denote the number of trivial A-modules appearing as factors in any given A-composition series of G. We prove the following

Theorem. *Let A be a finite cyclic group acting on the finite group G of odd order. Suppose that A normalizes a Sylow system of G. Then*

$$h(G) \leq 2\ell(A) + \mathbf{c}(G;A).$$

It is straightforward to show that $\mathbf{c}(G;A) = \ell(C_G(A))$ in case where G is solvable and $(|G|,|A|) = 1$. If A is nilpotent we shall see in Section 3 that $\mathbf{c}(G;A) = 0$ if and only if $C_G(A) = 1$. It follows that as an immediate consequence of the above theorem we have

Corollary. *Let A be a finite cyclic group acting fixed point freely on the finite group G of odd order. Then*

$$h(G) \leq 2\ell(A).$$

2. Main Tools

THEOREM 2.1. *Let A be a finite nilpotent group acting on a finite solvable group G and let V be a kGA-module for a field k such that V_G is homogeneous. If A normalizes a Sylow system of GA, then there is a homogeneous component of V_N which is stabilized by A for any A-invariant normal subgroup N of G.*

PROOF. □

PROPOSITION 2.2. *Let V be a faithful kA-module over a finite field k of characteristic s. A has a regular orbit on V if A is cyclic.*

PROOF. This follows from a result which is essentialy due to Dade and is stated as a proposition in [**4**], and Theorem 1.1 in [**14**]. □

THEOREM 2.3. *Let PQA be a finite group where P is a p-group and Q is a q-group for distinct primes p and q such that q is not a Fermat prime if $p=2$. Assume that $P \lhd PQA$, $Q \lhd QA$. Assume further that the following are satisfied:*
(a) A is cyclic;
(b) P is an extraspecial p-group for some prime p where $C_A(P)=1$ and $Z(P) \leq Z(PQA)$;
(c) Q/Q_0 is of class at most two and of exponent dividing q where $Q_0 = C_Q(P)$; and $1 = C_A(Q/Q_0)$.

Let χ be a complex PQA-character such that χ_P is faithful. Then χ_A contains the regular A-character.

PROOF. Similar to .... without the assumption of coprimeness. □

THEOREM 2.4. *Let GA be a finite group with $G \trianglelefteq GA$ where G is solvable, A is cyclic. Let P be a p-subgroup of G, for some prime p, such that $P/Z(P)$ is elementary abelian, $P \trianglelefteq GA$, $\Phi(P)=P'$, $exp(P)=p$ if p is odd, and $P/\Phi(P)$ is completely reducible as a GB-module for any subgroup B of A.*

Let Q be an A-invariant q-subgroup of $C_G(\Phi(P))$ for a prime q which is coprime to $p|A|$and not a Fermat prime if $p=2$. Assume that $Q/Q_0/Z(Q/Q_0)$ is elementary abelian where $Q_0 = C_Q(P)$, and $QC_G(P/P') \trianglelefteq GA$, and that $[Q,P]=P$ if $P' \neq 1$. Let $1 = C_G(Q/Q_0)$.

Assume further that the following hold:
(i) $P=[P,B]^G$ for every $B \leq A$ with $\ell(B) \geq 1$,
(ii) if P is nonabelian, $Q=[Q,C]^{N_G(Q)}Q_0$ for every $C \leq A$ with $\ell(C) \geq 2$,

Let χ be a complex GA-character such that $P \not\leq \ker(\chi)$. Then χ_A contains the regular A-character.

PROOF. Similar to .... without the assumption of coprimeness. □

3. PROOF OF THE THEOREM

Let A act on the group solvable group G. We denote the number of trivial A-modules appearing as factors in any A-composition series of G by $\mathbf{c}(G;A)$. More generally, for any normal A-series $1 = N_{k+1} < N_k < N_{k-1} < \cdots < N_1$ of G and A-invariant normal subgroups M_i for $i = 1,\ldots,k$ of G with $N_{i+1} \leq M_i < N_i$ and $P_i = N_i/M_i$, we write $\mathbf{c}(P_k,\ldots,P_1;A)$ for $\sum_{i=1}^{k} \mathbf{c}(P_i;A)$.

Let A act on G and normalize a Sylow system of G. Then by a slight modification of Lemma 8.2 of the same paper, one can show the existence of an irreducible A-tower in Turull's sense (see [**11**]), namely the existence of sections $P_i = S_i/T_i$, $i = 1,\ldots,h$, of G where S_i and T_i are subgroups of G such that $T_i \lhd S_i$ and $h = h(G)$ satisfying the following conditions:

(a) P_i is a nontrivial p_i-group, for some prime p_i,

(b) $\Phi(P_i) \leq Z(P_i), \Phi(\Phi(P_i)) = 1$ and if p_i is odd, then P_i has exponent p_i,

(c) P_i is A-invariant, for $i = 1,\ldots,h$,

(d) $p_i \neq p_{i+1}$, for $i = 1,\ldots,h-1$,

(e) $T_i = \mathrm{Ker}(S_i \text{ on } P_{i+1})$, for $i = 1,\ldots,h-1$,

(f) $T_h = 1$ and $S_h \leq F(G)$,

(g) $[\Phi(P_{i+1}),S_i] = 1$, for $i = 1,\ldots,h-1$,

(h) $(\prod_{1\leq j<i} S_j)A$ acts irreducibly on $\tilde{P}_i$.

We should also note that if A is a nilpotent group acting fixed point freely on the group G, then A is a Carter subgroup of the semidirect product GA having G as a normal complement, and hence Lemma 8.1 in [**3**] guarantees that A normalizes a Sylow system of G. Furthermore in this case we clearly have $\mathbf{c}(G;A) = 0$ which shows that the Corollary is an immediate consequence of the Theorem.

Now we proceed to the proof of Theorem. Since A normalizes a Sylow system of G, by the above remark we may assume the existence of an irreducible A-tower $P_1,\ldots,P_h$ with $P_i = S_i/T_i$ satisfying the conditions (a)-(h) for each $i = 1,\ldots,h$. Notice that it is sufficient to establish the following claim in order to complete the proof of the theorem.

Let A be a cyclic group and let $P_1,\ldots,P_h$ be a sequence of A-invariant sections satisfying the conditions $(a),(c),(d),(e)$ of a group G of odd order. Then $h \leq 2\ell(A) + \mathbf{c}(P_h,\ldots,P_1;A)$.

Let $\ell = \ell(A)$ and $h = h(G)$. By the above remark we may assume that $P_1,\ldots,P_h$ is an irreducible A-tower. Set $V = P_h, P = P_{h-1}, Q = S_{h-2}, X = PQS_{h-3}\ldots S_1$, and let χ be the XA-character afforded by V.

(1) *We can assume that χ is a complex character by the Fong-Swan theorem.*

(2) $Q = [Q,B]^{S_{h-3}\ldots S_1}Q_0$ for every $B \leq A$ with $\ell(B) \geq 1$, and hence $P = [P,B]^X$ for every $B \leq A$ with $\ell(B) \geq 1$.

PROOF. Let $B \leq A$ with $\ell(B) \geq 1$ such that $Q \neq [Q,B]^{S_{h-3}\dots S_1} Q_0$. Recall that the Frattini factor group of Q/Q_0 is $S_{h-3}\dots S_1 A$-irreducible. Hence $[Q,B] \leq \Phi(Q)Q_0$, that is $[P_{h-2}/\Phi(P_{h-2}),B] = 1$. It follows that $[P_i,B] = 1$ for each $i < h-2$. Then the sequence

$$P_{h-2}/\Phi(P_{h-2}), P_{h-3}, \dots, P_1$$

is an A-chain of length $h-2$ centralized by B so that the cyclic group A/B acts on each term of this chain. By induction assumption we have

$$h-2 \leq 2(\ell-1) + \mathbf{c}(P_{h-2}/\Phi(P_{h-2}), P_{h-3}, \dots, P_1; A)$$

which is impossible. Hence $Q = [Q,B]^{S_{h-3}\dots S_1} Q_0$ for every $B \leq A$ with $\ell(B) \geq 1$.

Next let $B \leq A$ with $\ell(B) \geq 1$ such that $P \neq [P,B]^X$. Set $P_1 = [P,B]^X$. Recall that the Frattini factor group of P is XA-irreducible. Hence $P_1 \leq \Phi(P)$. Now B is trivial on $P/\Phi(P)$. It follows that $[Q/Q_0,B] = 1$, which is not the case. □

(4) *Theorem follows.*

PROOF. We are now ready to apply Theorem 2.6 to the action of XA on V as P and Q satisfy the required hypothesis, and obtain the contradiction that χ_A contains the regular A-character, that is $C_V(A) \neq 0$. Since $C_V(A)$ is subnormal subgroup of $S_h \dots S_1 A$ we see that $\mathbf{c}(P_{h-1}\dots P_1;A) \leq \mathbf{c}(P_h \dots P_1;A) - 1$. Thus we have

$$h-1 \leq 2\ell + \mathbf{c}(P_h \dots P_1;A) - 1$$

which completes the proof. □

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Frobenius Kernel

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Abstract

Let G be a finite group and H be a non-trivial proper subgroup of G. The group G is called a Frobenius group if $H \cap H^g = 1$ for all $g \in G-H$. The set $K = (G - g \in G \bigcup H^g) \cup \{1\}$ is called the Frobenius kenel and H is called the Frobenius complement. Using character Theory it is proved that K is a normal subgroup of G. In this paper we present some group theoretical proofs that K is a subgroup of G under certain conditions.

Keywords:
Frobenius group; Frobenius complement; Frobenius kernel.

AMS Mathematics Subject Classification 2020:
20H20-20F50

1 Introduction

Let G be a finite group acting on a set Ω, $|\Omega| > 1$. Then G is called a Frobenius group if

(a) G acts transitively on Ω,

(b) $G_\alpha \neq 1$ for any $\alpha \in \Omega$,

(c) $G_\alpha \cap G_\beta = 1$ for all $\alpha, \beta \in \Omega$, $\alpha \neq \beta$.

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Let $H = G_\alpha$ for some $\alpha \in \Omega$, then for any $\beta \in \Omega$, the group G_β is conjugate to G_α, i. e. $G_\beta = G_\alpha^g = H^g$ for some $g \in G$.

Therefore $F = Gg \in G\bigcup H^g$ is the set of elements of G that don't fix any element of Ω. We set $K = F \cup \{1\} = (Gg \in G\bigcup H^g) \cup \{1\}$. The subgroup H is called a Frobenius complement and the set K is called the Frobenius kernel of G. It is easy to prove that $N_G(H) = H$ and that

$$|K| = |G| + 1 - (|H|)[G:H] - 1 = [G:H] = n$$

Therefore $|G| = |K||H|$ and $H \cap K = 1$.

An equivalent definition of a Frobenius group is the following: G is called a Frobenius group with complement H if $1 \neq HG$ and $H \cap H^g = 1$ for all $g \in GH$.

It is proved by G. Frobenius in 1901.
G. Frobenius, Über auflősbare Gruppen , IV., Berl. Ber., 1901 (1901), PP. 1216-1230.

That the Frobenius kernel K is a normal subgroup of G. The proof by Frobenius uses the theory of character. But since 1901 many attempts have been made to prove the normality of K without using character theory. Ofcourse K contains the unit element 1 and it is a normal subset of G, but the difficulty is to prove that K is closed under multiplication.

2 Character Theory

Forethere proofs of the normality of K in G can be found in the following refrences where character theory is used:
L. Dornhoff, Group Representation Theory, Part A: ordinary representation theory, Vol 7, Pure and Appl. Math, Marcel Dekker, Inc., New York (97)
W. Feit, On a conjecture of Frobenius, Proc. Amer. Math. Soc. 7(1956)177-187.
L. C. Grove, Groups and Characters, Pure and Appl. Math John Wiley and sons Inc. New York 1997.
M. Hall Jr, The theory of groups, The Macmillan company, New York, 1959.
B. Huppert, Endlichp Gruppen, Springer-Verlag, 1967.
W. Koapp and P. Schmid, A note on Frobenius groups, J. Group Theory, 12(2009) 393-400.

Assume that G is a Frobenius group with complement H and kernel K. We will present a character theoretic proof that K is a normal subgroup of G. This proof is a modification of the proof in:
W. Knapp and P. Schmid, A note on Frobenius groups, Journal of group theory 12(2009) 393-400.

Define the function $\psi : G \longrightarrow C$ by

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$$\psi(g) = \{ \mid H|,\ if \quad g \in K$$

0, otherwise

Since K is a normal subset of G, ψ is a class function on G. We will prove ψ is a character of G with $ker\psi = K$, proving KG.

Let $\chi \in Irr(G)$. We will show that

$$C_\chi = \langle \chi, \psi \rangle = 1|G|x \in G\sum\chi(x)\psi(x) = 1|G|x \in K\sum\chi(x)|H| = 1nx \in K\sum\chi(x)$$

is a non-negative integer.

If $\chi = 1_G$ the trivial character of G, then $(\chi, \psi) = 1$. So assume $\chi \neq 1_G$.

$G - K = g \in G\bigcup(H-1)^g$ is a disjoint union of n conjugates of $H-1$, hence

$$(\chi, 1_G) = 1|G|g \in G\sum\chi(g) = 1|H|h \in H - 1\sum\chi(h) + 1n|H|x \in K\sum\chi(x) = \\ (\chi_H, 1_H) - \chi(1)|H| + C_\chi|H| \Longrightarrow C_\chi = \chi(1) - |H|(\chi_H, 1_H)$$

is an integer and $|H| \mid C_\chi - \chi(1)$.

Therefore ψ equals a linear combinators of irreducible characters of G with integer coefficients.

Next we compute

$$1 = (\chi, \chi) = 1|H|h \in H - 1\sum|\chi(h)|^2 + 1|G|x \in K\sum|\chi(x)|^2$$

$1|H|h \in H - 1\sum|\chi(h)|^2 = (\chi_H, 1_H) - \chi(1)^2|H|$ is a non-negative rational number.

By Cauchy-Schwartz inequality $x \in K\sum|\chi(x)|^2 \geq 1n(x \in K\sum|\chi(x)|)^2$ with equality iff $|\chi(x)| = \chi(1)$ for all x.

Moreover $1nx \in K\sum|\chi(x)|^2 \geq C_\chi^2$ with equality iff $\chi(x) \in R$, with the same signe. Therefore

$$1 \geq ((\chi_H, \chi_H) - \chi(1)^2|H|) + C_\chi^2|H| \qquad (*)$$

But

$$C_\chi^2 - \chi(1)^2 = (C_\chi - \chi(1))(C_\chi + \chi(1))$$

is divisible by $|H| \Longrightarrow$ the right-hand side of $(*)$ is a positive integer and consequently we must have equality. Thus $C_\chi = 1nx \in K\sum|\chi(x)| = \chi(1)$ is the degree of χ and this proves that $ker\psi = KG$.

In the paper entiiled " Some properties of the finite Frobenius groups " published in Aut J. Math. and comput. 4(1)(2023)57-61, which was dedicated to Prof. J. Moori we obtained some character theoretic properties of the finite Frobenius groups as follows.

Recal that G is a Frobenius group with complement H and kernel K, $G = KH$, $K \cap H = 1$, if K is a normal subgroup of G, $n = [G : H]$.

G acts on the set of cosets of H as a transitive permutation group of degree n and the number of orbits of H on this set is called the rank of G and is denoted by s.

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Proposition 2.1. Let χ be the permutation character of H acting on K by conjugation. Then $\chi = s\rho_H + 1_H$ where ρ_H and 1_H are the regular and the identity character of H.

Proposition 2.2. Let G be a Frobenius group with kernel K as a subset. If all elements of K commute, then K is a normal subgroup of G.

Proof. K is a normal subset of G with identity and G acts on it by conjugation. Let η be the permutation character associated with this action. For $g \in G$, $\eta(g)$ is the number of $k \in K$ such that $k^g = g^{-1}kg = k$. We have $\eta(1) = |K|$ and if $g \neq 1$, and $k^g = k$, then $k \in C_G(g) \cap K = C_K(g)$. Since $G = Kg \in G \cup H^g$ we distinguish the following cases.

(I) $1 \in K, k^g = k \Longrightarrow k \in C_K(g) \Longrightarrow \eta(g) = |C_K(g)|$

(II) $1 \neq g \in g \in G \bigcup H^g \Longrightarrow g$ belongs to some conjugate of H

Some we may take
$g \in H \Longrightarrow k^g = k \Longrightarrow g = g^k \in H \cap H^k \Longrightarrow k \in H \cap K = 1 \Longrightarrow \eta(g) = 1.$

Therefore

$$\eta(g) = \{ \mid K|, \; if \quad g = 1,$$
$$1, \text{ if } 1 \neq g \in \{H^x \mid x \in G\},$$

$|C_K(g)|, \; if \quad 1 \neq g \in K$. By assumption all elements of K cummute, hence

$$\eta(g) = \{ \mid K|, \; if \quad g \in K,$$

1, if $1 \neq g \in \{H^x \mid x \in G\}$. Now we see that $ker\eta = KG$. □

3 Group Theory

As we mentioned earlier no group theory proof exists for the Frobenius kernel to be a subgroup, but in some special cases there is a proof that we will present here.

If G is a Frobenius group with complement H and kernel K, then

$$N_G(H) = H$$

and $|K| = [G : H]$.

Lemma 3.1. If NG, $G = NH$, $N \cap H = 1$, then $N \leq K$.

Lemma 3.2 (Burnside). If G is a finite group and P is a Sylow p-subgroup of G such that $N_G(P) = C_G(P)$, then P has a normal complement in G, i. e. , there is NG such that $G = NP$, $N \cap P = 1$.

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Theorem 3.3. Let G be a Frobenius group with complement H and kernel K. Assume that H is an abelian p-group. Then K is a normal subgroup of G.

Proof. From the fact that $N_G(H) = H$ and the fact that H is abelian we obtain

$$H = N_G(H) \geq C_G(H) \geq H$$

Therefore $N_G(H) = C_G(H)$. But $(|H| : [G : H]) = 1$ from which it follows that H is a Sylow p-subgroup of G. Now by Burnside's theorem H has a normal complement N in G, i. e. $G = NH$, $N \cap H = 1$ and NG. □

Corollary 3.4. Let G be a finite Frobenius group with complement H and kernel K. Suppose H is centralized by a Sylow p-subgroup of G. Then KG.

Proof. By assumption $H \leq C_G(P)$ where P is a Sylow p-subgroup of G. But:

$$1 \neq x \in H \Longrightarrow C_G(x) \leq H$$

Therefore $C_G(P) \leq H \Longrightarrow C_G(P) = H = N_G(P)$. Now by Burnside's theorem: $\exists NG, G = NP. \Longrightarrow |N| = [G : P]$.

By Lemma 1,

$$N \leq K \Longrightarrow |N| = [G : P] \leq |K| = [G : H]. \Longrightarrow |P| \geq |H| \Longrightarrow P = H$$

By Theorem 3 $\Longrightarrow K \leq G$. □

Another group theoretical proof under different conditions exists that we mention below. If $2 \mid |H|$, there is an elementary proof that $K \leq G$ due to Bender:

Let t be an element of order 2 in H and $g \in G \backslash H$. Then either

$$a = t \cdot g^{-1} t g = t t^g = [t, g]$$

is in K or $\exists x \in G$ such that $1 \neq a \in H^x$. If $a \in H^x \Longrightarrow a \in H^x \cap H^{xt} \cap H^{xt^g}$

Because $a^t = a^{-1} = a^{tg} \Longrightarrow H^x = H^{xt} = H^{xt^g}, t, t^g \in H^x$

If $H^x = H$ contradicts $t \in H$ and $t^g \notin H$.

Therefore: $tt^g \in K$ if $g \in G \backslash H$.

Let $\{g_1, \cdots, g_n\}$ be a transversal of H in G, $n = [G : H]$.

$tt^{g_i} = tt^{g_j} \Longleftrightarrow t^{g_i} = t^{g_j} \Longleftrightarrow t^{g_i g_j^{-1}} = t \Longleftrightarrow g_i g_j^{-1} \in H$

The elements $tt^{g_1}, \cdots, tt^{g_n}$ are pairwise distinct. $\Longrightarrow K = \{t^{g_1} t, \cdots, t^{g_n} t\}$

Now we show that $K \leq G$. For $t^{g_i t}$ there exists g_s such that $t^{g_i t} = tt^{g_s}$.

$\Longrightarrow (tt^{g_i})(tt^{g_j}) = t(t^{g_i} t) t^{g_j} = t(tt^{g_s}) t^{g_j} = t^{g_s} t^{g_j} = (tt^{g_i g_s^{-1}}) g_s \in K^{g_s} = K$

$tt^g \in K$ for $g \in G \backslash H$.

If the complement H is solvable, then K is a subgroup of G.

R. H. Shaw, Remarks on a theorem of Frobenius, Proc. Amer. Math. Soc., 3(1952) 970-972.

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H acts on $K-\{1\}$ by conjugation without a fixed point, orbits of size $|H|$. Therefore $|H| \mid |K|-1 \Longrightarrow (|H|,|K|)=1$. If $K \leq G$, then K is a Hall-subgroup of G. Also H is a $Hall-subgroup$ of G.

Looking at the Frobenius group G as a transitive permutation group on the set Ω, $|\Omega|=n$, $H=G_\alpha$, $\alpha \in \Omega$. Then $|\Omega|=[G:H]$. The number of orbits of H on Ω is called the rank of G, $r=rank(G)$.

Each nontrivial H-orbit has six $|H|$ and there are $s=n-1|H|$ such orbits, $rank(G)=1+s=1+n-1|H|$.

If $rank(G) \leq 3$, then $K \leq G$ by using elementary group theory.

W. Knapp and P. Schmiott, Frobenius groups of low rank, Acch Math. 117(2021) 121-127.

The proof uses the fact that H is a Hall-subgroup of G, and for every prime p dividing $|K|=n=[G:H]$, the Sylow p-subgroup of G are contained in K. Thus for small rank, consequence of the Sylow theorem implies $K \leq G$. In particular if $[G:H]$ is n prime power then K is a Sylow subgroup of G.

4 Properties of the Frobenius Kernel

Suppose G is a Frobenius group with complement H and kernel K. Assume KG, $G=HK$.

G has a unique kernel. If K is solvable, then H is nilpotent. Thompson showed that K is always nilpotent. Any subgroup of H of order p^2 or pq is cyclic p,q primes. If $P \in Syl_p(H)$, $p \neq 2$, then P is cyclic and $p=2$, P is cyclic or generalized quaternion. K has an automorphism without fixed points, if $|H|$ is even, then K is abelian.

Example that K is non-abelian.

D_{2n}, n odd is Frobenius with kernel of order 2.

Praglandan Perumal: Msc. Thesis

$V_2(5):SL_2(5)$

$GF(q)^*$ acts by right multiplication on $(GF(q),+)$. The corresponding semidirect product $GF(q)^*(q)$ is Frobenius group.

Classifying quotients of the Highwater algebra

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(Joint work with Clara Franchi (Catholic University of the Sacred Heart, Milan) and Mario Mainardis (University of Udine))

Abstract

Axial algebras are a class of non-associative algebras with a strong natural link to groups and have recently received much attention. They are generated by axes which are semisimple idempotents whose eigenvectors multiply according to a so-called fusion law. Of primary interest are the axial algebras with the Monster type (α, β) fusion law, of which the Griess algebra (with the Monster as its automorphism group) is an important motivating example.

By previous work of Yabe, and Franchi and Mainardis, any symmetric 2-generated axial algebra of Monster type (α, β) is either in one of several explicitly known families, or is a quotient of the infinite-dimensional Highwater algebra $\mathscr{H}$, or its characteristic 5 cover $\hat{\mathscr{H}}$. We complete this classification by explicitly describing the infinitely many ideals and thus quotients of the Highwater algebra (and its cover). As a consequence, we find that there exist 2-generated algebras of Monster type (α, β) with any number of axes (rather than just $1, 2, 3, 4, 5, 6, \infty$ as we knew before) and of arbitrarily large finite dimension.

In this talk, we do not assume any knowledge of axial algebras.

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Keywords:
axial algebras, finite simple groups, Monster group, Jordan algebras, baric algebras.

AMS Mathematics Subject Classification 2020:
20D08, 17C27, 17D99

Introduction and overview of results

Recently several finite simple groups, such as 3-transposition groups and many of the sporadic groups including the Monster, have been realised as automorphism groups of axial algebras of Monster type (α, β). In fact, the originating example of an axial algebra is the Griess algebra, which has the Monster sporadic simple group as its automorphism group. The Griess algebra turns out to be an axial algebra of Monster type $(\frac{1}{4}, \frac{1}{32})$. Almost all Jordan algebras are axial algebras of Monster type and Matsuo algebras, which are related to 3-transposition groups, are also examples. There is a hope that a full classification of these algebras will lead to a more unified approach to (a large portion of) finite simple groups, including the sporadics.

As for many algebraic structures (semisimple Lie algebras being one notable example), an understanding of the structure of these algebras requires a full classification of the 2-generated objects. The primitive 2-generated axial algebras of Monster type $(\frac{1}{4}, \frac{1}{32})$ were first classified by Ivanov, Pasechnik, Seress, and Shpectorov in [8], extending earlier work of Norton [10] (for the Griess algebra) and Sakuma [13] (for certain types of OZ-type vertex operator algebras of central charge $1/2$). These algebras are known as Norton-Sakuma algebras and fall into nine isomorphism classes.

It was Rehren in [11, 12] who first introduced the generalisation to Monster type (α, β) and began a systematic study of the 2-generated algebras. He concentrated on the *symmetric* algebras - those which admit an automorphism switching the two generators. He generalised the eight Norton-Sakuma algebras to eight families of examples. Implicit in his analysis is a case division into a *generic* case and two *critical* cases $\alpha = 2\beta$ and $\alpha = 4\beta$ – this is made explicit by Franchi, Mainardis and Shpectorov in [4]. Joshi introduced some new families of Monster type $(2\beta, \beta)$ (the critical case $\alpha = 2\beta$) in [9, 6]. In an unexpected development, Franchi, Mainardis, and Shpectorov in [3], and independently Yabe in [14], found the infinite-dimensional 2-generated Highwater algebra $\mathscr{H}$, which is of Monster type $(2, \frac{1}{2})$ (and falls into the other critical case).

A major breakthrough came from Yabe, who gave in [14] an almost complete classification of the symmetric 2-generated primitive axial algebras of Monster type in characteristic other than 5. The remaining case was considered by Franchi and Mainardis in [1], who introduced a characteristic 5 cover $\hat{\mathscr{H}}$ of the Highwater algebra and showed that all the cases not included in Yabe's classification are factors of $\hat{\mathscr{H}}$. Putting these all together we have the following:

Theorem 1. [14, 1] *A symmetric* 2*-generated primitive axial algebra of Monster type* (α,β) *is isomorphic to one of the following:*

1. *a* 2*-generated primitive axial algebra of Jordan type* α*, or* β*;*
2. *a quotient of* $\mathscr{H}$*, or* $\hat{\mathscr{H}}$ *in characteristic* 5*;*
3. *one of the algebras in a family listed in* [14, Table 2].

The 2-generated primitive axial algebras of Jordan type were classified by Hall, Rehren and Shpectorov in [7] and are of dimension at most 3. Every algebra in case (3) above is known and of dimension at most 8. In contrast, the Highwater algebra $\mathscr{H}$ and its cover $\hat{\mathscr{H}}$ have infinite dimension.

In this talk, we describe our classification of the quotients of the Highwater algebra $\mathscr{H}$ and of its characteristic 5 cover $\hat{\mathscr{H}}$, which completes the above classification. Moreover, it will turn out that our classification will give explicit bases for the ideals and hence also their quotients.

In our paper [2], we proceed by defining a covering algebra $\hat{\mathscr{H}}$ for all characteristics (not just 5). In characteristic 5, this coincides with the cover of the Highwater algebra and is of Monster type, but for other characteristics, it has a larger non-Monster type fusion law. However, this object allows us to provide a unified proof of our results. In the talk, we will give a simplified version of this story, concentrating on the Highwater algebra itself and not worrying about our cover.

We define a tuple $(\alpha_0,\dots,\alpha_D)\in\mathbb{F}^{D+1}$ to be of *ideal-type* if $\alpha_0\neq 0\neq\alpha_D$, $\sum_{i=0}^{D}\alpha_i=0$, and there exists $\varepsilon=\pm1$ such that $\alpha_i=\alpha_{D-i}$ for all i. With this combinatorial definition, we can state our main theorem concisely.

Theorem 2. [2] *For every* $D\in\mathbb{N}$*, there is a bijection between the set of ideal-type* $(D+1)$*-tuples* $(\alpha_0,\dots,\alpha_D)\in\mathbb{F}^{D+1}$*, up to scalars, and the set of minimal ideals of axial codimension* D *of* $\hat{\mathscr{H}}$ *given by*

$$(\alpha_0,\dots,\alpha_D)\mapsto\left(\sum_{i=0}^{D}\alpha_i a_i\right).$$

In particular, every ideal of $\mathscr{H}$ is principal.

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Graphs and probabilities defined on finite groups

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Abstract

In this expository talk, we discuss various probabilities of finite groups (e.g. commuting probability, relative commuting probability, g-commuting probability, autocommuting probability, cyclicity degree etc.) and their relations with various graphs defined on groups (such as commuting graph, relative commuting graph, g-non-commuting graph, cyclic graph etc.).
Keywords:
Finite group; Commuting Probability; Commuting graph; Graphs and Probability.

AMS Mathematics Subject Classification 2020:
20D60, 20P05,05C25

1 Introduction

Let G be a finite group. The commuting probability of G, denoted by $\Pr(G)$, is the probability that a randomly chosen pair of elements of the group commute. Thus

$$\Pr(G) = \frac{|\{(x,y) \in G \times G : xy = yx\}|}{|G|^2}.$$

Starting from Erdös and Turán [11], many mathematicians have considered commuting probability and its generalizations over the years. Some well-known generalizations of

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$\Pr(G)$ are relative commuting probability (denoted by $\Pr(H,G)$ and introduced by Erfanian, Rezaei and Lescot [12]), g-commuting probability (denoted by $\Pr_g(G)$ and introduced by Pournaki and Sobhani [17]), etc.

The relative commuting probability of G subject to a given subgroup $H \leq G$ is defined by

$$\Pr(H,G) = \frac{|\{(x,y) \in H \times G : xy = yx\}|}{|H||G|}.$$

The g-commuting probability of G subject to a given element $g \in G$ is defined by

$$\Pr_g(G) = \frac{|\{(x,y) \in G \times G : [x,y] = g\}|}{|G|^2},$$

where $[x,y]$ denotes the commutator of x and y, that is $xyx^{-1}y^{-1}$. Note that $\Pr(H,G) = \Pr(G) = \Pr_g(G)$ if $H = G$ and $g = 1$, the identity element of G.

Das and Nath [6] generalize the notions of $\Pr(H,G)$ and $\Pr_g(G)$ by considering the following ratio

$$\Pr_g(H,K) = \frac{|\{(x,y) \in H \times K : [x,y] = g\}|}{|H||K|},$$

where $H,K \leq G$ and $g \in G$. Note that $\Pr_g(H,K) = \Pr_g(G)$ if $H = K = G$; and $\Pr_g(H,K) = \Pr(H,G)$ if $K = G$ and $g = 1$. Further generalizations of $\Pr_g(G)$ can be found in [15, 7]. The notions of $\Pr(G)$ and $\Pr_g(G)$ are also generalized through group actions and groups of automorphisms (see [19, 9, 10]).

Certain probabilities analogous to commuting probability, that we are going to discuss, are cyclicity degree (denoted by $\Pr_C(G)$ and introduced by Patrick, Sherman, Sugar and Wepsic [16]), nilpotency degree (denoted by $\Pr_N(G)$ and introduced by Dubose-Schmidt, Galloy and Wilson [8]) and solvability degree (denoted by $\Pr_S(G)$ and introduced by Fulman, Galloy, Sherman and Vanderkam [13]). These probabilities are defined as

$$\Pr_C(G) := \frac{|\{(x,y) \in G \times G : \langle x,y \rangle \text{ is cyclic}\}|}{|G|^2},$$

$$\Pr_N(G) := \frac{|\{(x,y) \in G \times G : \langle x,y \rangle \text{ is nilpotent}\}|}{|G|^2}$$

and

$$\Pr_S(G) := \frac{|\{(x,y) \in G \times G : \langle x,y \rangle \text{ is solvable}\}|}{|G|^2}.$$

Another popular topic, after the work of Brauer and Fowler [5], is the study of graphs defined on groups motivated by commutativity. Let G be a finite non-abelian group with centre $Z(G)$. The commuting graph of G, denoted by $\mathscr{C}(G)$, is a simple undirected graph whose vertex set is $G \setminus Z(G)$ and two distinct vertices x and y are adjacent if $xy = yx$. The complement of $\mathscr{C}(G)$, denoted by $\mathscr{N}\mathscr{C}(G)$, is known as the non-commuting graph of G.

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For any subgroup H of G, define $Z(H,G) = \{x \in G : xy = yx$ fo all $y \in H\}$. The relative non-commuting graph of a subgroup H of G, denoted by $\mathcal{NC}(H,G)$ and introduced by Tolue and Erfanian [20], is a simple undirected graph whose vertex set is $G \setminus Z(H,G)$ and two distinct vertices x and y are adjacent whenever x or y in H and $xy \neq yx$. For $g \in G$, the g-noncommuting graph of G (denoted by $\mathcal{NC}_g(G)$ and introduced by Tolue, Erfanian and Jafarzadeh [21]) is a simple undirected graph whose vertex set is G and two distinct vertices x and y are adjacent whenever $[x,y] \neq g, g^{-1}$. The notions of $\mathcal{NC}(H,G)$ and $\mathcal{NC}_g(G)$ are further generalized by relative g-noncommuting graph. The relative g-noncommuting graph of a subgroup H of G, denoted by $\mathcal{NC}_g(H,G)$ and introduced by Sharma and Nath [18], is a simple undirected graph whose vertex set is G and two distinct vertices x and y are adjacent whenever x or y in H and $[x,y] \neq g, g^{-1}$. Note that $\mathcal{NC}_g(H,G) = \mathcal{NC}_g(G)$ if $H = G$ and $\mathcal{NC}_g(H,G) = \mathcal{NC}(H,G)$ if $g = 1$.

Certain graphs analogous to commuting graphs, that we are also going to discuss, are cyclic graph (denoted by $\mathcal{EP}(G)$ and introduced by Abdollahi and Hassanabadi [2]), nilpotent graph (denoted by $\mathcal{N}(G)$ and introduced by Abdollahi and Zarrin [3]) and solvable graph (denoted by $\mathcal{S}(G)$ and introduced by Hai-Reuven [14]).

Let X be a property (cyclic, abelian, nilpotent, solvable etc.) of a finite group G. We define $X(G) := \{x \in G : \langle x,y \rangle$ is a X-group for all $y \in G\}$. Clearly, if X represents "abelian" then $X(G) = Z(G)$. We write $\mathrm{Cyc}(G)$, $\mathrm{Nil}(G)$ and $\mathrm{Sol}(G)$ to denote $X(G)$ if X represents "cyclic", "nilpotent" and "solvable" respectively. A graph is called X graph of G if the vertex set is $G \setminus X(G)$ and two distinct vertices x and y are adjacent if $\langle x,y \rangle$ is a X-group. Thus, if X represents "cyclic", "nilpotent" and "solvable" then X graph of G is nothing but $\mathcal{EP}(G)$, $\mathcal{N}(G)$ and $\mathcal{S}(G)$ respectively. The complement of X graph is called non-X graph of G.

2 Main results

In this talk, we discuss various results on the probabilities and graphs defined in the previous section. However, we would like give emphasis on the following results that establish connections among various probabilities and graphs defined on finite groups. We write $|e(\mathcal{G})|$ to denote the number of edges in a graph $\mathcal{G}$.

Theorem 2.1. *[1] The number of edges in $\mathcal{C}(G)$ and $\mathcal{NC}(G)$ are given by*

$$2|e(\mathcal{C}(G))| = |G|^2 \Pr(G) + |Z(G)|(|Z(G)|+1) - |G|(2|Z(G)|+1)$$

and

$$2|e(\mathcal{NC}(G))| = |G|^2(1 - \Pr(G)).$$

Theorem 2.2. *[20] The number of edges in $\mathcal{NC}(H,G)$ is given by*

$$|e(\mathcal{NC}(H,G))| = |H||G|(1 - \Pr(H,G)) - \frac{|H|^2}{2}(1 - \Pr(H)).$$

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Theorem 2.3. *[21] The number of edges in $\mathscr{NC}_g(G)$ is given by*

$$2|e(\mathscr{NC}_g(G))| = \begin{cases} |G|^2(1-\Pr(G)), & \text{if } g=1 \\ |G|^2(1-\Pr_g(G))-|G|, & \text{if } 1 \neq g \in K(G) \text{ and } g^2=1 \\ |G|^2(1-2\Pr_g(G))-|G|, & \text{if } 1 \neq g \in K(G) \text{ and } g^2 \neq 1 \\ |G|^2-|G|, & \text{if } g \neq K(G), \end{cases}$$

where $K(G)=\{[x,y] : x,y \in G\}$.

Theorem 2.4. *[18] Let $|e(\mathscr{NC}_g(H,G))|$ be the number of edges in $\mathscr{NC}_g(H,G)$ and $K(H,G) := \{[x,y] : x \in H, y \in G\}$.*

(a) If $g \notin K(H,G)$ then $2|e(\mathscr{NC}_g(H,G))| = 2|H||G|-|H|^2-|H|$.

(b) If $g=1$ then

$$2|e(\mathscr{NC}_g(H,G))| = 2|H||G|(1-\Pr_g(H,G))-|H|^2(1-\Pr_g(H)).$$

(c) If $g \in K(H,G)$, $g \neq 1$ and $g^2=1$ then $2|e(\mathscr{NC}_g(H,G))|$

$$= \begin{cases} 2|H||G|(1-\Pr_g(H,G))-|H|^2(1-\Pr_g(H))-|H|, & \text{if } g \in H \\ 2|H||G|(1-\Pr_g(H,G)_-|H|^2-|H|, & \text{if } g \in G \setminus H. \end{cases}$$

(d) If $g \notin K(H,G)$, $g \neq 1$ and $g^2 \neq 1$ then $2|e(\mathscr{NC}_g(H,G))|$

$$= \begin{cases} 2|H||G|(1-\sum\limits_{u=g,g^{-1}}\Pr_u(H,G)) \\ \quad -|H|^2(1-\sum\limits_{u=g,g^{-1}}\Pr_u(H))-|H|, & \text{if } g \in H \\ 2|H||G|(1-\sum\limits_{u=g,g^{-1}}\Pr_u(H,G))-|H|^2-|H|, & \text{if } g \in G \setminus H. \end{cases}$$

Theorem 2.5. *The number of edges in $\mathscr{EP}(G)$ and $\mathscr{NEP}(G)$ (the complement of $\mathscr{EP}(G)$) are given by*

$$2|e(\mathscr{EP}(G))| = |G|^2\Pr_C(G)+|\operatorname{Cyc}(G)|(|\operatorname{Cyc}(G)|+1)-|G|(2|\operatorname{Cyc}(G)|+1)$$

and

$$2|e(\mathscr{NEP}(G))| = |G|^2(1-\Pr_C(G)).$$

Theorem 2.6. *The number of edges in $\mathscr{N}(G)$ and $\mathscr{NN}(G)$ (the complement of $\mathscr{N}(G)$) are given by*

$$2|e(\mathscr{N}(G))| = |G|^2\Pr_N(G)+|\operatorname{Nil}(G)|(|\operatorname{Nil}(G)|+1)-|G|(2|\operatorname{Nil}(G)|+1)$$

and

$$2|e(\mathscr{NN}(G))| = |G|^2(1-\Pr_N(G)).$$

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Theorem 2.7. *[4] The number of edges in $\mathscr{S}(G)$ and $\mathscr{N}\mathscr{S}(G)$ (the complement of $\mathscr{S}(G)$) are given by*

$$2|e(\mathscr{S}(G))| = |G|^2\Pr_S(G) + |\operatorname{Sol}(G)|(|\operatorname{Sol}(G)|+1) - |G|(2|\operatorname{Sol}(G)|+1)$$

and

$$2|e(\mathscr{N}\mathscr{S}(G))| = |G|^2(1-\Pr_S(G)).$$

We conclude this talk by mentioning analogous notions of these probabilities and graphs defined on other algebraic structures.

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On monomial Isaacs π-partial characters of π-separable groups

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Abstract

Let G be π-separable group for some set π of primes. The element g of G is said π-element if every prime divisor of the order of g lies in π. In other words, the order of g is π-number. We denote G^0 the set of π-elements of G. Also, we say that classes of G which π-elements lie are the π-classes.
Let χ be a complex character. We denote χ^0 the restriction of χ to G^0. A complex-valued function φ on G^0 is said to be an Isaacs π-partial character of G if $\varphi = \chi^0$ for some character χ of G. In other words, we could say Isaacs π-partial character of G is a class function of G^0 which lifts to a complex character (not necessarily unique) of G. An Isaacs π-parial character of G is irreducible if it cannot be written as a sum of two Isaacs π-partial characters and denote $\mathrm{I}_\pi(G)$ the set of all irreducible Isaacs π-partial characters of G.

It is obvious that $\mathrm{Irr}(G) = \mathrm{I}_{\pi(\mathrm{G})}(G)$. Alao, let $p^{'}$ be the complement of the singleton set $\{p\}$ relative to $\pi(G)$. If $\pi = p^{'}$ for prime p, then G is p-solvable and by Fong-Swan Theorem, $\mathrm{IBr_p}(G) = \mathrm{I_{p'}(G)}$. A character $\chi \in \mathrm{Irr}(G)$ is said to be π-special if its degree and its order are π-numbers and, for any subnormal subgroup M of G and any irreducible constituent θ of χ_M, the order of θ is a π-number. We denote $\mathscr{X}_\pi(G)$ as the set of all π-special characters of G. $\mathrm{B}_\pi(G)$ is a subset of $\mathrm{Irr}(G)$ constructed in some unambiguous

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way which is too complex to explain here, and having the property that each member $\varphi \in \mathrm{I}_\pi(G)$ has a unique lift in this subset. Roughly speaking, every $\chi \in \mathrm{B}_\pi(G)$ is in the form $\chi = \gamma^G$, where γ is a π-special character of a subgroup of G. As a result, $\mathscr{X}_\pi(G) \subseteq \mathrm{B}_\pi(G)$. Let $\varphi = \chi^0$ be a Isaacs π-partial character. We define $\ker\varphi = \ker\chi$ such that $\chi \in \mathrm{B}_\pi(G)$ and $\varphi = \chi^0$ also $\bigcap_{\varphi \in \mathrm{I}_\pi(G)} \ker\varphi = O_{\pi'}(G)$. We can also induce Isaacs π-partial characters. Suppose θ is a Isaacs π-partial character of $H \subseteq G$. We define θ^G as an Isaacs π-partial character of G using the usual formula for the induced character but applied only for π-elements of G. We refer [1] for more information on Isaacs π-partial characters.

We will talk about the solvability of π-separable groups that every irreducible Isaacs π-partial character is monomial which we call M_π-group and also we obtain some results on degrees of monomial Isaacs π-partial character degrees. We will try to generalize Taketa Theorem, Ito-Michler Theorem and Thompson Theorem for Isaacs π-partial characters.

Keywords:
Isaacs π-partial character; monomial character; π-separable group; solvable group.

AMS Mathematics Subject Classification 2020:
20C20-20B15

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A graph associated to central automorphisms in a finite group

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Abstract

Let G be a finite group and A_G be the automorphism group of G. We associated a bipartite graph, denoted by Γ_{G,A_G}, to G and its automorphism group as follows: two parts of the vertex set are $G \setminus L(G,Z(G))$ and $\mathrm{A}_G \setminus \mathrm{Aut}_c(G)$, where $L(G,Z(G))$ is the set of elements $g \in G$ such that $g^{-1}\alpha(g) \in Z(G)$, for all $\alpha \in \mathrm{A}_G$ and $\mathrm{Aut}_c(G)$ is the central automorphism group of G which is the set of automorphisms $\beta \in \mathrm{A}_G$ in which $g^{-1}\beta(g) \in Z(G)$, for all $g \in G$. Two vertices $g \in G \setminus L(G,Z(G))$ and $\alpha \in \mathrm{A}_G \setminus \mathrm{Aut}_c(G)$ are adjacent if and only if $g^{-1}\alpha(g) \notin Z(G)$. In this article, we investigate some basic properties of this graph and state some conjectures and open problems at the end.

Keywords:
Bipatite graph; central automorphism; diameter; girth; planar graph.

AMS Mathematics Subject Classification 2020:
20P05

[1]Speaker.

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1 Introduction

The study of algebraic structures, using the properties of graph theory, tends to an exciting research topic in the last decade. In this case, there are many papers on assigning a graph to a group and investigating of algebraic properties of group through the associated graph. Some relevant research can be found [1, 2, 3, 4, 5, 6]. In this paper, we are going to assign a new graph to a finite group G and its automorphism group $\mathrm{Aut}(G)$. Let G be a finite group and $\mathrm{A}_G = \mathrm{Aut}(G)$. We set

$$L(G,Z(G)) = \{g \in G : g^{-1}\alpha(g) \in Z(G) \text{ for all } \alpha \in \mathrm{A}_G\}. \tag{1}$$

Clearly, $L(G,Z(G))$ is a subgroup of G. Also,

$$\mathrm{Aut}_c(G) = \{\alpha \in \mathrm{A}_G : g^{-1}\alpha(g) \in Z(G) \text{ for all } g \in G\}, \tag{2}$$

stands as the central automorphism of G which is a normal subgroup of A_G. We define a graph denoted by Γ_{G,A_G} as a bipartite graph with a vertex set consisting two parts $G \setminus L(G,Z(G))$ and $\mathrm{A}_G \setminus \mathrm{Aut}_c(G)$. Two vertices $g \in G \setminus L(G,Z(G))$ and $\alpha \in \mathrm{A}_G \setminus \mathrm{Aut}_c(G)$ are adjacent if and only if $g^{-1}\alpha(g) \notin Z(G)$. It is clear that Γ_{G,A_G} is a simple graph and if G is an abelian group then the graph is null graph. Thus we always assume that G is non-abelian. Moreover, if $\mid \mathrm{A}_G \setminus \mathrm{Aut}_c(G) \mid = 1$ then Γ_{G,A_G} is a null graph. Hence, we may suppose $\mathrm{Aut}_c(G)$ is a proper subgroup of A_G.

2 Main Results

Some of the main results are stated here and the rest will be stated in the talk.

Lemma 2.1. The degree of every vertex of the graph Γ_{G,A_G} is at least 2.

Theorem 2.2. The girth of the graph Γ_{G,A_G} is 4.

Lemma 2.3. Let $g \in G \setminus L(G,Z(G))$ and $\alpha \in \mathrm{A}_G \setminus \mathrm{Aut}_c(G)$, then we have

(i) If g is adjacent to an automorphism α_i, for some $1 \leqslant i \leqslant s$, then g will be adjacent to every element in $\alpha_i \mathrm{Aut}_c(G)$.

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(*ii*) If α is adjacent to an element x_i, for some $1 \leqslant i \leqslant r$, then α will be adjacent to every element in $L(G,Z(G))x_i$.

Corollary 2.4. Every right coset of $G \setminus L(G,Z(G))$ and every left coset of $\mathrm{A}_G \setminus \mathrm{Aut}_c(G)$ form a complete bipartite subgraph or empty bipartite subgraph in graph Γ_{G,A_G}.

Lemma 2.5. Let $g \in G \setminus L(G,Z(G))$ and $\alpha \in \mathrm{A}_G \setminus \mathrm{Aut}_c(G)$ be arbitrary vertices. Then

(*i*) $\deg(\alpha) \geq [|G| \setminus |L(G,Z(G))|2]+1$,

(*ii*) $\deg(g) \geq [|\mathrm{A}_G| \setminus |\mathrm{Aut}_c(G)|2]+1$,

note that $[x]$ stands for the integer part of real number x.

Theorem 2.6. $\mathrm{diam}(\Gamma_{G,\mathrm{A}_G}) \leq 4$.

Theorem 2.7. Let $t = \min\{|G \setminus L(G,Z(G))|, |\mathrm{A}_G \setminus \mathrm{Aut}_c(G)|\}$. Then there exists a cycle of length $2n$ in Γ_{G,A_G}, for $n = 2,3,\ldots,t$.

Theorem 2.8. Let $\alpha(\Gamma_{G,\mathrm{A}_G})$ be an independence number of the graph Γ_{G,A_G}. Then $\alpha(\Gamma_{G,\mathrm{A}_G}) = \max\{|G \setminus L(G,Z(G))|\ ,\ |\mathrm{A}_G \setminus \mathrm{Aut}_c(G)|\}$.

Lemma 2.9. Let $L(G,Z(G))$ be a maximal subgroup of the finite group G. Then the graph Γ_{G,A_G} is the completed bipartite graph.

Lemma 2.10. If $|\mathrm{Aut}_c(G)| \geq 3$ and $|L(G,Z(G))| \geq 3$, then Γ_{G,A_G} is not planar.

Lemma 2.11. Let $|\mathrm{Aut}_c(G)| \geq 3$ and $|G \setminus L(G,Z(G))| \geq 2$. If there exists $g \in G \setminus L(G,Z(G))$ such that $g \neq g^{-1}$, then Γ_{G,A_G} is not planar.

Corollary 2.12. Let $|\mathrm{Aut}_c(G)| \geq 3$. If $|G \setminus L(G,Z(G))| \geq 2$ or $|L(G,Z(G))| \geq 2$, then Γ_{G,A_G} is not outer planar,.

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Transitive q-ary designs and q-ary graphs

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Abstract

In 1976, Delsarte introduced the notion of q-analogs of designs, and q-analogs of graphs were introduced recently by M. Braun, M. De Boeck, V. Mikulić Crnković, A. Švob and the author. In this talk we give a method for constructing transitive q-analogs of designs and graphs. Transitive q-analogs of designs and graphs yield a special kind of constant dimension subspace codes, called orbit codes.

Keywords:
q-ary design; q-ary graph; transitive group; subspace code.

AMS Mathematics Subject Classification 2020:
05B05-05E18

1 Introduction

In [7], Delsarte introduced the notion of q-analogs of designs. Let F_q^v be a vector space of dimension v over a finite field F_q. A subspace of vector space F_q^v of dimension k will be called a k-subspace. A (simple) t-$(v,k,\lambda;q)$ subspace design consists of a set $\mathscr{B}$ of k-subspaces of F_q^v, called blocks, such that each t-subspace of F_q^v is contained in exactly λ blocks.

[1]**Online Speaker**

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In 1987, Thomas (see [12]) constructed the first non-trivial q-analogs of design with parameters 2-$(n,3,7;2)$, $n>6, n=6k+1$ or $n=6k-1$. Further, Suzuki constructed 2-$(n,3,q^2+q+1;q)$ design, $n>6, n=6k+1$ or $n=6k-1$ (see [10, 11]), and in [9], Miyakawa, Munemasa, and Yoshiara gave a classification of 2-designs for which $n\in\{6,7\}$, $q\in\{2,3\}$, $k=3$, having an automorphism group that acts transitively on the 1-subspaces.

Due to applications of q-analogs of designs to network coding and distributed storage, the interest in this subject has been renewed recently (see [3]). One of the most important recent result was given in [2], where the authors constructed a design over a finite field with parameters 2-$(13,3,1;2)$ which was the first known example of Steiner q-design that does not arise from spreads. Further, in [4] the authors apply difference methods for obtaining designs over finite fields.

Recently, q-analogs of graphs were introduced and q-ary strongly regular graphs were studied (see [1]). An undirected graph (without loops) on v vertices is a subset E of 2-dimensional subspace of the vector space F_q^v. The elements of E are called edges. We will present a method of constructing transitive q-ary designs and q-ary graphs.

2 Main results

A method of constructing primitive designs and graphs is given in [8]. The following generalization of the method by Key and Moori is given in [5].

Theorem 2.1. *Let G be a finite permutation group acting transitively on the sets Ω_1 and Ω_2 of size m and n, respectively. Let $\alpha\in\Omega_1$ and $\Delta_2=\bigcup_{i=1}^{s}\delta_iG_\alpha$, where $G_\alpha=\{g\in G\mid \alpha g=\alpha\}$ is the stabilizer of α and $\delta_1,\dots,\delta_s\in\Omega_2$ are representatives of distinct G_α-orbits on Ω_2. If $\Delta_2\neq\Omega_2$ and*

$$\mathscr{B}=\{\Delta_2 g : g\in G\},$$

then $\mathscr{D}(G,\alpha,\delta_1,\dots,\delta_s)=(\Omega_2,\mathscr{B})$ is a 1-$(n,|\Delta_2|,\frac{|G_\alpha|}{|G_{\Delta_2}|}\sum_{i=1}^{s}|\alpha G_{\delta_i}|)$ design with $\frac{m\cdot|G_\alpha|}{|G_{\Delta_2}|}$ blocks. The group $H\cong G/\bigcap_{x\in\Omega_2}G_x$ acts as an automorphism group on $(\Omega_2,\mathscr{B})$, transitively on points and blocks of the design.

If $\Delta_2=\Omega_2$ then the set $\mathscr{B}$ consists of one block, and $\mathscr{D}(G,\alpha,\delta_1,\dots,\delta_s)$ is a design with parameters 1-$(n,n,1)$. Suppose that the group G acts transitively on the points and blocks of a simple t-(v,k,λ) design. Then the design can be obtained as described in Theorem 2.1. If $\Omega_1=\Omega_2$ and Δ_2 is a union of self-paired and mutually paired orbits of G_α, then the design $\mathscr{D}(G,\alpha,\delta_1,\dots,\delta_s)$ is a symmetric self-dual design and the incidence matrix of that design is the adjacency matrix of a $|\Delta_2|-$regular graph.

In Theorem 2.2, given in [6], we present a method for constructing q-analogs of 1-designs having an automorphism group that acts transitively both on the 1-subspaces of the vector space F_q^v and on the set of blocks of the subspace designs.

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Theorem 2.2. *Let $G < GL(v,q)$ be a group acting transitively on the set of 1-subspaces of the vector space F_q^v. Let P be a subgroup of the group G and $\Delta = \bigoplus_{i=1}^{s} \delta_i P$ for distinct 1-spaces $\delta_1, ..., \delta_s$. Then $\mathscr{B} = \{\Delta g \mid g \in G\}$ is a set of blocks of a q-analog of design with parameters 1-$\left(v, \dim\Delta, \frac{|G|(q^k-1)}{|G_\Delta|(q^v-1)}; q\right)$ and $\frac{|G|}{|G_\Delta|}$ blocks. The group G acts as an automorphism group on the constructed q-analog of design, transitively on the 1-subspaces of the vector space F_q^v and on the set of blocks of the design.*

If $G < PGL(v,q)$, then G acts faithfully on the 1-subspaces of the vector space F_q^v. If the group G acts transitively on the points and blocks of a simple design t-$(v,k,\lambda;q)$, then the design can be obtained as described in Theorem 3.

The following theorem gives us the method of constructing transitive q-ary graphs.

Corollary 2.3. *Let $G < GL(v,q)$ acts transitively on the set of 1-subspaces of the vector space F_q^v. Further, let P be the stabilizer of a 1-dimensional subspace for that action and $\Delta = \bigoplus_{i=1}^{s} \delta_i P$ such that $\langle \Delta g \rangle = \langle \Delta g^{-1} \rangle$, $\forall g \in G$, and $P = G_\Delta$. If $\mathscr{B} = \{B_1, ..., B_b\}$ is the set of blocks of the q-analog of 1-design with the base block Δ, then $\mathscr{B}$ is a set of spaces of neighbours of a $(k-1)$-regular q-graph.*

Subspace codes are a class of codes used for random network coding. They are defined as sets of subspaces of some given ambient space F_q^v of dimension v over the finite field F_q. If all the codewords have the same dimension k, the subspace code is called a constant dimension code. A special class of constant dimension codes are orbit codes. Orbit codes are defined as orbits of a subgroup of the general linear group of order v over F_q. By applying Theorem 2.2 or Corollary 2.3, one obtains an orbit code.

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Group theoretic constructions of normal covers of the complete bipartite graphs $\mathbf{K}_{2^n,2^n}$.

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Abstract

It is thirty years since I introduced the concept of a normal quotient of a finite 2-arc-transitive graph, showing that each such graph is a normal cover of a 'basic' 2-arc-transitive graph. In the years since then a lot of progress has been made in identifying families of basic graphs. In particular, Cai Heng Li [2] identified all the basic 2-arc transitive graphs of prime power order, building on my joint work with Ivanov [1], and Li posed the problem of characterising their 2-arc-transitive normal covers which also had prime power order: he stated that he was '*inclined to think that non-basic* 2*-arc-transitive graphs of prime power order would be rare and hard to construct*'. This was the motivation for the joint work which I will report on.

Our work focused on 2-arc-transitive normal covers of one of these basic families, namely the complete bipartite graphs $\mathbf{K}_{2^n,2^n}$. We first proved that such graphs are usually Cayley graphs, and that all exceptions had to be based on a special family of groups called 'mixed dihedral groups'. We studied these mixed dihedral groups further and used them to build

- a new infinite family of 2-geodesic-transitive normal Cayley graphs which are neither distance-transitive nor 2-arc-transitive [3];

[1] Speaker.

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- a new infinite family of locally 2-arc-transitive semisymmetric graphs, (that is, provably not vertex-transitive), of 2-power order [4];
- a single 2-arc-transitive normal cover of $\mathbf{K}_{2^4,2^4}$ which is not a Cayley graph; it has order 2^{53}. We do not know if similar graphs exist as covers of $\mathbf{K}_{2^n,2^n}$, with $n > 4$ [5].

Keywords:
2-arc-transitive graph; normal cover; Cayley graph; edge-transitive; 2-groups

AMS Mathematics Subject Classification 2020:
20B25, 05C38

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Probability associated with the verbal subgroup of a finite group

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Abstract

To measure the commutativity of two elements in a group, we can define a commutator by $[x,y]=x^{-1}y^{-1}xy$ for all $x,y \in G$. Two elements, x and $y \in G$, commute if and only if $[x,y]=e$, where e is the identity element of G. Then, we want to determine the probability of two elements x and y in a finite group such that $[x,y]=e$. By using the free group, we can define a probability associated with a verbal subgroup of G denoted as $P_{G,w}(g)$. Up until now, there are many open problems about the structure of $P_{G,w}(g)$ and its implication for the underlying group structure. One of them is Amit's Conjecture which says the value of $P_{G,w}(e)$ never be less than $1/|G|$ for every finite nilpotent group G. This paper proves Amit's Conjecture for any words over two variables. As an application, we give a bound for the number of edges of the non-braid graph of any finite nonabelian groups and also bounds for some topological indices.

Keywords:
commutator of a group; probability of group elements; number of edges.

AMS Mathematics Subject Classification 2020:
60B15 , 20F05 , 05C25 , 05C09

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1 Introduction

The concept of the probabilities of two elements in a finite commutative group was first discussed by Paul Erdös and Turan [2], where the probability of picking $x, y \in G$, where G is a finite group, such that $xy = yx$ can be expressed as $\frac{k(G)}{|G|}$ where $|G|$ represents the order of G and $k(G)$ represent the number conjugacy classes of G. By defining the commutator of two elements in G, namely $[x,y] = x^{-1}y^{-1}xy$ then the above probability can be rewritten as the probability of getting $x, y \in G$ such that $[x,y] = e$ where e is the identity element in G.

What if the expression $[x,y]$ is replaced with another expression? By reviewing the elements of the free group generated by k variables, namely

$$w(x_1, x_2, ..., x_k)$$

then we can define a verbal mapping on the finite group, namely

$$\begin{array}{ccc} f_w : G^{(k)} & \to & G \\ (g_1, g_2, ..., g_k) & \mapsto & w(g_1, g_2, .., g_k) \end{array}.$$

Then the probability of having k-tuple $\overline{g} \in G^{(k)}$ so that $w(\overline{g}) = e$, is

$$\frac{|N_{G,w}(e)|}{|G|^k},$$

where $N_{G,w}(e) = \{\overline{g} \in G^{(k)} : w(\overline{g}) = e\}$. This probability is denoted by $P_{G,w}(e)$.
The problem of finding a lower bound of $P_{G,w}(e)$ is still an open problem that has not been solved until now. One of the unresolved problems raised by Amit (see [1]), concerns the lower bound of $P_{G,w}(e)$ in the nilpotent group as $P_{G,w}(e) \geq \frac{1}{|G|}$ for each word w. In this paper, we will give the lower bound of $P_{G,w}(e)$ for w an element of the free group with rank 2, for any finite group G.

2 Main results

Let G be a finite group. Let F_n denotes the free group generated by $S = \{x_1, x_2, \cdots, x_n\} \subseteq G$ and we write elements of F_n as $w(x_1, x_2, ..., x_n)$.

Suppose $w \in F_2$, the function $d(w, x_i)$ represents the sum of the powers of the component x_i in w and it is defined as the degree of w in x_i.

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Example : Let $w(x_1,x_2) = x_1x_2^{-1}x_1^2x_2^4 \in F_2$ then $d(w,x_1) = d(w,x_2) = 3$.

We define $N = \{w \in F_2 : d(w,x_1) = d(w,x_2)\}$. Note that by parsing the exponent of an element w in F_2 then $d(w,x_i)$ can be viewed as the number of components x_i in w minus the number of components x_i^{-1} in w.

Lemma 2.1. *For every group G and $w \in N$, $|N_{G,w}(e)|$ is divisible by $|G|$.*

Define for each $A = (a_{jk}) \in GL(2,\mathbb{Z})$ a homomorphism which maps x_i to $x_1^{a_{i1}}x_2^{a_{i2}}$ and w^A is the image of $w \in F_2$.

Example : Suppose $w = x_1x_2$ and $A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$ then $w^A = x_1x_2^2x_1^3x_2^5$.

Lemma 2.2. *Let F' be the commutator subgroup of F_2. If $A,B \in GL(2,\mathbb{Z})$ and $w \in F_2$ then*

$$(w^A)^B F' = w^{AB} F'.$$

Lemma 2.3. *There is a set Δ which generates $GL(2,\mathbb{Z})$ so that if $A \in \Delta$ and $w \in F_2$ then*

$$|N_{G,w^A}(e)| = |N_{G,w}(e)|$$

Lemma 2.4. *If $w \in F_2$ then there is $A \in GL(2,\mathbb{Z})$ such that $w^A \in N$.*

Theorem 2.5. *For every word $w \in F_2$, $|N_{G,w}(e)|$ is divisible by $|G|$.*

Proof. Suppose $w \in F_2$. There is $A \in GL(2,\mathbb{Z})$ such that $w^A \in N$. Since $GL(2,\mathbb{Z}) = \langle \Delta \rangle$ then A can be written as $A = A_1A_2...A_r$ for some $A_1,A_2,...,A_r \in \Delta$. Then there exits $a \in F'$ such that

$$w^A a = ((w^{A_1})^{A_2}.....)^{A_r}$$

Since $F' \subseteq N$ then $w^A a \in N$, hence $|N_{G,w^A a}(e)|$ is divisible by $|G|$. Based on Lemma 2.3 we have

$$|N_{G,w}(e)| = |N_{G,w^{A_1}}(e)| = |N_{G,(w^{A_1})^{A_2}}(e)| = ... = |N_{G,w^A a}(e)| \equiv 0 \ (mod\ |G|)$$

Therefore $|N_{G,w}(e)|$ is divisible by $|G|$. □

Corollary 2.6. *Suppose G is a finite group, then for every $w \in F_2$ we have*

$$P_{G,w}(e) \geq \frac{1}{|G|}.$$

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Some Results on Topological Indices of Graphs Associated to Groups and Rings

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Abstract

A topological index is a numerical value associated with a chemical compound that provides information about its molecular structure and properties. Researches on topological indices are initially related to graphs obtained from chemical structures to predict the biological activities and reactivity. Recently, the research on this topic has evolved on graphs in general and even on graphs obtained from algebraic structures, such as groups, rings or modules. This paper will present various researches and results on topological indices of graphs associated to groups, including the non-commuting graph and the co-prime graph, and a graph associate to rings, namely the non-zero divisor graph.

Keywords:
Topological index; Mathematical chemistry; Graph theory; Group theory; Ring theory.

AMS Mathematics Subject Classification 2020:
05C07; 05C09; 05C12; 05C90

[1]**Online Speaker**

1 Introduction

Topological indices provide numerical descriptors that capture important structural features of molecules, and serve as powerful tools for the analysis and prediction of various physicochemical properties and biological activities. The significance of topological indices lies in their ability to transform complex molecular structures into numerical representations, enabling the development of computational models and the efficient exploration of chemical space for various applications in drug discovery, materials science, and reaction chemistry [1].

One of the primary applications of topological indices is in the prediction of properties related to the biological activity of compounds, such as drug-likeness, toxicity, and bioactivity [2]. By analyzing the connectivity and arrangement of atoms within a molecule, topological indices can provide insights into how a compound interacts with biological targets, aiding in the design and optimization of new drugs. In addition to drug discovery, topological indices find applications in various other areas of mathematical chemistry and materials science. For example, they can be used to predict physical properties of molecules, such as boiling points, solubilities, and partition coefficients, aiding in the selection of appropriate solvents or understanding the behavior of compounds in different environments [3].

The first type of topological index has been discovered by Wiener [4] in 1947, in which the concept of Wiener number considering the path in a graph is introduced. In [4], the Wiener number of some paraffins are determined and their boiling points are also predicted. Then, Hosoya [5] reformulated the formula of Wiener number, known as Wiener index of a graph, $W(\Gamma)$, and its formula is given in the following.

$$W(\Gamma) = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} d(i,j),$$

where $d(i,j)$ is the distance between vertices i and j, and m is the total number of vertices in a graph Γ.

Since then, various types of topological indices have been developed based on either chemistry or mathematical perspectives. In 1972, Gutman and Trinajstić [6] introduced the degree-based topological index, Zagreb index, which is divided into two types; first Zagreb index, M_1, and second Zagreb index, M_2, defined as follows.

$$M_1(\Gamma) = \sum_{v\in v(\Gamma)} (\deg(v))^2$$

and

$$M_2(\Gamma) = \sum_{\{u,v\}\in E(\Gamma)} \deg(u)\deg(v).$$

In addition, the Szeged and Harary indices are given in the following definitions.

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Definition 1.1. ***[7] The Szeged Index***
Let Γ be a simple connected graph with vertex set $V(\Gamma) = \{1, 2, \ldots, n\}$. The Szeged index, $Sz(\Gamma)$ is given as in the following :

$$Sz(\Gamma) = \sum_{e \in E(\Gamma)} n_1(e|\Gamma) n_2(e|\Gamma),$$

where the summation embraces all edges of Γ,

$$n_1(e|\Gamma) = |\{v | v \in V(\Gamma), d(v,x|\Gamma) < d(v,y|\Gamma)\}|$$

and

$$n_2(e|\Gamma) = |\{v | v \in V(\Gamma), d(v,y|\Gamma) < d(v,x|\Gamma)\}|$$

which means that $n_1(e|\Gamma)$ counts the Γ's vertices are closer to one edge's terminal x than the other while $n_2(e|\Gamma)$ is vice versa.

Definition 1.2. ***[8] The Harary Index***
Let Γ be a connected graph with vertex set $V = \{1, 2, \ldots, n\}$. Half the elements' sum in the reciprocal distance matrix, $D^r = D^r(\Gamma)$, is what is known as the Harary index, written as

$$H = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} D^r(i, j),$$

where
$D^r(i,j) = \begin{cases} \frac{1}{d(i,j)} & if \quad i \neq j, \\ 0 & if \quad i = j, \end{cases}$ *and $d(i, j)$ is the shortest distance between vertex i and j.*

The Randić index is a graph-theoretical descriptor that quantifies the complexity or branching structure of a molecular graph. It was introduced by Milan Randić [9] in 1975 and has found applications in various fields of chemistry. The Randić index of a molecular graph is calculated based on the topological distances between pairs of vertices (atoms) in the graph. It is defined as the sum of the reciprocal square roots of the product of the degrees of connected pairs of vertices, written as

$$R(\Gamma) = \sum_{u,v \in E(\Gamma)} \frac{1}{\sqrt{deg(u)deg(v)}}.$$

Then, the Randić index is modified and introduced a concept of general zeroth order Randić index, which is defined as

$${}^0R_\alpha = \sum_{u \in V(\Gamma)} (deg(u))^\alpha,$$

where α can be any non-zero real number [10]. In 2018, the general inequalities of ${}^0R_\alpha$ is determined, as stated in the following.

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Theorem 1.1. *[11] Let α be a positive integer and Γ be any graph, then*

$$^0R_\alpha \leq \frac{1}{c}(^0R_{\frac{1}{\alpha}}(\Gamma))^\alpha$$

holds for $c = 1$ whereas the inequailty is not always true for $c > 1$. The equality sign in the above inequality holds if and only if $\alpha > 1$ or Γ has size zero.

Recently, in 2020, a new topological index, Sombor index has been established by Gutman [12]. The Sombor index of a graph, $SO(\Gamma)$, is defined as follows.

$$SO(\Gamma) = \sum_{u,v \in E(\Gamma)} \sqrt{\deg(u)^2 + \deg(v)^2}.$$

In this paper, some results on the topological indices of some graphs associated to groups and rings are presented.

2 Topological Indices of Graphs Associated to Groups

The study on topological indices have received attention from many researchers, from multidisciplinary field. In this paper, some results on both degree and distance based topological indices of some graphs of groups are stated.

Following theorem presents the Wiener index of some graphs associated to some finite groups. The graphs that include in this paper are the non-commuting graph and coprime graph.

Theorem 2.1. *[15] Let G be the generalised quaternion group, Q_{4n} of order $4n$ where $n \geq 2$, Γ_G is the non-commuting graph of G and $W(\Gamma_G^{NC})$ is the Wiener index of Γ_G. Then,*

$$W(\Gamma_G^{NC}) = 2n(5n-7)+6.$$

In addition, the coprime graph of a group G is denoted as Γ_G^{CO} and its Wiener index has been determined by Alimon et al [17] in 2020.

Theorem 2.2. *[17] Let G be the dihedral group of order $2n$, where $n \geq 3$. Then, if n is an odd prime, $W(\Gamma_G^{CO}) = 3n^2 - 3n + 1$. Meanwhile, if $n = 2^k, k \in Z^+$, then $W(\Gamma_G^{CO}) = (2n-1)^2$.*

Next, the first and second Zagreb index of the non-commuting graph for some finite groups, denoted as $M_1(\Gamma_G^{NC})$ and $M_2(\Gamma_G^{NC})$, respectively, are stated in the following theorems.

Theorem 2.3. *[16] Let G be a dihedral group, D_{2n} of order $2n$ where $n \geq 3$. Then,*

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$$M_1(\Gamma_G^{NC}) = \begin{cases} n(5n-4)(n-1) & \text{if} \quad n \text{ is odd}, \\ n(5n-8)(n-2) & \text{if} \quad n \text{ is even}, \end{cases}$$

and

$$M_2(\Gamma_G^{NC}) = \begin{cases} 2n(n-1)^2(2n-1) & \text{if} \quad n \text{ is odd}, \\ 4n(n-2)^2(n-1) & \text{if} \quad n \text{ is even}. \end{cases}$$

Theorem 2.4. *[15] Let G be the generalised quaternion group, Q_{4n} of order $4n$ where $n \geq 2$. Then,*

$$M_1(\Gamma_G^{NC}) = 8n(5n^2 - 9n + 4),$$

and

$$M_2(\Gamma_G^{NC}) = 32n(2n^3 - 5n^2 + 4n - 1).$$

Theorem 2.5. *[17] Let G be a dihedral group, D_{2n} and Γ_G^{CO} is coprime graph of G. Then, if $n = 2^k$, where $k \in Z^+$, the Szeged index of coprime graph for D_{2n} is as follows :*

$$Sz(\Gamma_G^{CO}) = 4n^2 - 4n + 1.$$

Furthermore, the Harary index of the non-commuting graph associated to the dihedral groups are stated in the next theorem.

Theorem 2.6. *[18] Let G be a dihedral group, D_{2n} and Γ_G^{NC} is a non-commuting graph of G. Then,*

$$H(\Gamma_G^{NC}) = \begin{cases} \frac{1}{4}[(n-2)(7n-3)+n] & \text{if} \quad n \text{ is even}, \\ \frac{1}{4}[(n-1)(7n-2)] & \text{if} \quad n \text{ is odd}. \end{cases}$$

In 2023, Roslly et al. [19] determined the general formula of Randić index of the non-commuting graph associated to three finite groups, namely the dihedral groups, the generalised quaternion groups and the quasidihedral groups, as presented in Theorems 2.8, 2.9 and 2.10, respectively.

Theorem 2.7. *[19] Let G be a dihedral group, D_{2n} and Γ_G^{NC} is a non-commuting graph of G. Then,*

$$R(\Gamma_G^{NC}) = \begin{cases} \frac{n\sqrt{2n(n-1)}+4n(n-1)}{4\sqrt{2n(n-1)}} & \text{if} \quad n \text{ is odd}, \\ \frac{n\sqrt{2n(n-2)}+4n(n-2)}{4\sqrt{2n(n-2)}} & \text{if} \quad n \text{ is even}. \end{cases}$$

Theorem 2.8. *[19] Let G be the generalised quaternion group, Q_{4n} and Γ_G^{NC} is a non-commuting graph of G. Then,*

$$R(\Gamma_G^{NC}) = \frac{4n(n-1)}{\sqrt{8n(n-1)}} + \frac{n}{2}.$$

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Theorem 2.9. *[19] Let G be the quasidihedral group, QD_{2^n} and Γ_G^{NC} is a non-commuting graph of G. Then,*

$$R(\Gamma_G^{NC}) = \frac{2^{n-1}(2^{n-1}-2)}{\sqrt{(2^{n-1})(2^n-4)}} + \frac{2^{n-1}(2^{n-2}-1)}{2^n-4}.$$

Some results on Sombor index of the non-commuting graph related to groups are found by Khasraw et al. [13].

Theorem 2.10. *[13] Let G_1 be the dihedral groups, G_2 be the generalised quaternion groups and G_3 be the quasidihedral groups. Then,*

$$SO(\Gamma_{G_1}^{NC}) = \begin{cases} n(n-1)\left[\sqrt{2}(n-1)+\sqrt{4(n-1)^2+n^2}\right] & \text{if } n \text{ is odd}, \\ n(n-2)\left[\sqrt{2}(n-2)+\sqrt{4(n-2)^2+n^2}\right] & \text{if } n \text{ is even}, \end{cases}$$
$$SO(\Gamma_{G_2}) = 8n(n-1)\left[\sqrt{2}(n-1)+\sqrt{4(n-1)^2+n^2}\right],$$
$$SO(\Gamma_{G_3}) = \sqrt{2}(2^n-4)(2^{2n-3}-2^{n+1}) + (2^{2n-2}-2)\sqrt{(2^n-4)^2+2^{2n-2}}.$$

3 Topological Indices of Graphs Associated to Rings

In this section, some results on the topological indices of zero divisor graph for some commutative rings, found by [21] and [22] are presented.

Theorem 3.1. *[21] The first Zagreb index of zero divisor graph for Z_{p^k} is $2(p^{k-1} - p^k)\left(k-1+\lceil\frac{k-1}{2}\rceil\right) + (p^k+1)(p^{k-1}-1) + 3\left(p^{\lceil\frac{k-1}{2}\rceil}-1\right)$ where $k \geq 3$ for $p = 2$ and $k \geq 2$ for odd prime p.*

Theorem 3.2. *[22] The general zeroth-order Randić index of the zero divisor graph for the ring $Z_{p^k q}$ when $\alpha = 1$,*

$$R_1^0 = k(p^k - p^{k-1})(2q-1) - p^k - p + 2 - \left(\frac{p^k - p^{\lceil\frac{k+3}{2}\rceil}}{p^{\lceil\frac{k+1}{2}\rceil}}\right).$$

4 Conclusion

In this paper, some results on a few types of topological indices of some graphs associated to some finite groups and commutative rings are stated. These theoretical results can benefit other disciplines in predicting the chemical and physical properties of the molecules.

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5 Acknowledgement

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Post-quantum Blockchains using hash functions using higher dimensional special linear groups over finite fields as platforms

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Abstract

We define new families of Tillich-Zémor hash functions, using higher dimensional special linear groups over finite fields as platforms. The Cayley graphs of these groups combine fast mixing properties and high girth, which together give rise to good preimage and collision resistance of the corresponding hash functions. We justify the claim that the resulting hash functions are post-quantum secure.
Keywords:
Group-based Cryptography; Block-chain; Special Linear Groups; Tillich-Zemor Hash Function; Expander Graphs.

AMS Mathematics Subject Classification 2020:
20xxx-20yyy

1 Blockchain and Hash Functions

It is projected that by 2025, approximately 10% of the worldwide gross domestic product (GDP) will be stored using blockchain technology. Blockchains are digital tools that employ cryptography techniques to safeguard information against unauthorized alterations.

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The foundation of the Bitcoin cryptocurrency relies on this technology. Blockchain-based products find applications across various sectors such as finance, manufacturing, and healthcare, contributing to a market valued at over US $150 billion. Functioning as a secure digital record or ledger, a blockchain is collectively maintained by users worldwide, eliminating the need for a central administration.

To design such a Blockchain network, we need to construct the ”blocks” which contain the information about the transaction and unique value to identify this block; known as a “Hash”.

We define new families of Tillich-Zémor hash functions, using higher dimensional special linear groups over finite fields as platforms. The Cayley graphs of these groups combine fast mixing properties and high girth, which together give rise to good preimage and collision resistance of the corresponding hash functions. We use a theorem by Arzhantseva and Biswas (2018) concerning the expanding properties of the Cayley graphs of these groups. We justify the claim that the resulting hash functions are post-quantum secure.

This is a joint work with C. Le Coz, C. Battarbee, R. Flores, T. Koberda.

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Abstracts

of

Speakers in Parallel Sessions

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(p,q,r)-generations and the conjugacy classes ranks for a finite group

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(Joint work with Dr ABM Basheer and Prof. TT Seretlo)

Abstract

A finite group G is called (l,m,n)-*generated*, if it is a quotient group of the triangle group $T(l,m,n) = \langle x,y,z | x^l = y^m = z^n = xyz = 1 \rangle$. In [2], Moori posed the question of finding all the (p,q,r) triples, where p, q and r are prime numbers, such that a non-abelian finite simple group G is a (p,q,r)-generated.

Let X be a conjugacy class of a finite group G. The *rank* of X in G, denoted by *rank*$(G:X)$, is defined to be the minimum number of elements of X generating G. We investigate the ranks of the non-identity conjugacy classes of the above three mentioned finite simple groups. The Groups, Algorithms and Programming (GAP) [1] and the Atlas of finite group representatives [3] are used in our computations.

Keywords:
(p,q,r)-generations; Conjugacy classes ranks; generation; simple groups; structure constants.

AMS Mathematics Subject Classification 2020:
20xxx-20yyy

[1] Speaker.

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Generalized Z-fuzzy soft β-covering based rough matrices and its application

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Abstract

Fuzzy sets, rough sets and soft sets are different mathematical tools mainly developed to deal with uncertainty. The combination of these theories has a wide range of applications in decision analysis. In this paper, we defined a generalized Z-fuzzy soft β-covering-based rough matrices. Some algebraic properties are explored for this newly constructed matrix. The main aim of this paper is to propose a novel MAGDM model using generalized Z-fuzzy soft β-covering-based rough matrices. A MAGDM algorithm based on AHP method is created to recruit the best candidate for an assistant professor job in an institute and a numerical example is presented to demonstrate the created method.

Keywords:
β-level soft set, Fuzzy soft β-adhesion, Generalized Z-fuzzy soft β-covering based rough matrix.
AMS Mathematics Subject Classification 2020:
03E72, 90B50

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Constructing some designs invariant under alternating groups.

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Abstract

In this study, we construct some $1-$designs that are invariant under the alternating group A_n (for $n \geq 5$). The method we use is due to J Moori [1] and is called the fixed point Method. Using this method, J Moori and A Saeidi in [2] and [3] constructed designs and codes invariant under the alternating groups A_n for a maximal subgoup of index $n-1$. In this talk we use the fixed point method to construct designs invariant under the alternating group A_n with its maximal subgroup of index n.

Keywords:
Fixed Points ; Alternating Group; Design.

AMS Mathematics Subject Classification 2020:
05B05

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On the Parallel and Descriptive Complexities of Group Isomorphism via Weisfeiler–Leman

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Abstract

The GROUP ISOMORPHISM problem takes as input two finite groups G and H, and asks if $G \cong H$. When the groups are given by their multiplication tables, GROUP ISOMORPHISM is strictly easier than GRAPH ISOMORPHISM under several notions of parallel reduction, including for instance AC^0-reductions (Chattopadhyay, Torán, & Wagner, *ACM Trans. Comput. Theory*, 2013). Despite this fact, there are several measures of complexity, including logical definability (descriptive complexity) and the parallel (circuit) complexity of isomorphism testing, that are cornerstones of the GRAPH ISOMORPHISM literature but have received minimal attention in the setting of groups. In this talk, I will discuss recent advances in both of these directions using the Weisfeiler–Leman algorithm for groups (Brachter & Schweitzer, LICS 2020).

Keywords:
Group Isomorphism, Graph Isomorphism, Weisfeiler–Leman, Descriptive Complexity, Circuit Complexity.

AMS Mathematics Subject Classification 2020:
20-08, 68Q17, 68Q19, 68Q25, 20A15

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Finite groups with many complemented subgroups

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Abstract

All groups considered are finite.

Throughout this abstract G stands for a finite group. A subgroup H of G is said to be *complemented* in G if there exists a subgroup K of G such that $G = HK$ and $H \cap K = 1$. Such a subgroup K of G is called a *complement* to H in G. A number of authors have examined the structure of G under the assumption that certain subgroups are complemented in G. For example P. Hall proved that a group G is soluble if and only if every Sylow subgroup is complemented in G. In [4] the same author also proved that a group G is supersoluble with elementary abelian Sylow subgroups if and only if every subgroup of G is complemented in G. He called such groups *complemented*. The class of all complemented groups is closed under taking subgroups and under taking epimorphic images.

In [2] A. Ballester-Bolinches and X. Guo proved that the class of all complemented groups is just the class of all groups G for which every minimal subgroup is complemented in G. In [3] the same authors and K.P. Shum denoted the family of complemented groups by $\mathscr{BNS}$ and they established the structure of minimal non-$\mathscr{BNS}$-groups (minimal non-complemented groups). In [8] we give the classification of non-soluble groups all of whose second maximal subgroups are complemented groups.

If G is the Suzuki group $Sz(8)$, then every nontrivial proper subgroup of G is a non-complemented subgroup of G. Clearly, there exists a soluble group with the

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same property. If a finite p-group G has exactly one subgroup of order p, then every proper subgroup of G is a non-complemented subgroup of G. It is well-known that a p-group with a unique subgroup of order p is cyclic or a generalized quaternion group. We will show that a non-soluble group has exactly 30 non-complemented subgroups if and only if it is isomorphic to the alternating group A_5 and a group with less than 30 non-complemented subgroups is soluble.

Keywords:
complemented subgroups; complemented groups.

AMS Mathematics Subject Classification 2020:
20D10-20D20

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Error correcting codes from 2-representation of unitary group U(3,3)

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Abstract

In this thesis, we use modular repesenation theoretic to find error correcting codes admit the unitary group $U(3,3)$ as a primitive permutation group. We show that every binary linear code admiting $G = U(3,3)$ as a primitive permutation group is a submodule of the permutation module of the primitive action of G. Our aim is to find the set of all linear codes from the set of 2-representations of a finite simple group. Then explain that according to Theorem 2.2 of[1], if the Schur multiplier of the group G is trivial and P is a permutation module of degree n, then every binary linear code of length n invariant under G is a submodule of P. We will find the irreducible submodules of the simple group in our case the group will be unitary group $U(3,3)$. We will be using magma[2] to find all those submodules. After finding the submodules we will find the maximal subgroups, each maximal subgroup has a corresponding permutation module we will also find them using magma.

We find submodule of these permutation modules, these submodules are codes from the $U(3,3)$, We Classify some codes and determine properties of some binary codes such as the minimum distance and other properties that we described in the first part such as the weight, minimum weight and the support. We will be able to tell whether the certain code is good error correcting or error detecting

[1]Nyikadzino TG.

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code based on properties they possess.
We determine support designs from these binary codes using codewords of minimum weight. In order to obtain block designs we must determine the set of points, make a list of all possible base blocks then solve the problem of huge number of constructed designs with the same parameters. We establish the linkages between some linear codes and designs from primitive groups.

Keywords:
Code; Design; Permutation module; Permutation representation; Premitive action.

AMS Mathematics Subject Classification 2020:
20xxx-20yyy

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Normal supercharacter theories of the dicyclic groups

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Abstract

Supercharacter theory of finite groups was developed by Diaconis and Isaacs as a natural generalization of the classical ordinary character theory of finite groups.Supercharacter theories of many finite groups such as the cyclic groups, the dihedral groups, the Frobenious groups, etc. were well studied by several authors. In this paper we consider the dicyclic group and find the normal supercharacter theories of this groups in special cases.

Keywords:
Character theory; Supercharacter; Lattice of normal subgroups.

AMS Mathematics Subject Classification 2020:
20C12-20E15

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Hulls of Negacyclic Codes Over $\mathbb{F}_q + v\mathbb{F}_q$

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Abstract

Let $\mathscr{R} = \mathbb{F}_q + v\mathbb{F}_q$, where $q = p^m$, p is prime number and $v^2 = v$. Clearly $\mathscr{R} \simeq \mathbb{F}_q[v]/ < v^2 - v >$ is a non chain ring with q^2 elements. In this work, we study hulls of negacyclic codes over $\mathscr{R}$. Firstly, we give the characterization of the hull of negacyclic codes in terms of their generator polynomials with respect to the Euclidean inner product over the finite ring $\mathscr{R}$. We also introduce the hull dimensions of negacyclic code over $\mathscr{R}$. Furthermore, the enumeration of negacyclic codes over $\mathscr{R}$ with hulls of a fixed dimension is determined. We establish an alternative simpler expression of the average $q-$dimensions of the Euclidean hulls of negacyclic codes $\mathtt{E}_{\mathscr{R}}(n,-1)$ over finite non chain rings $\mathscr{R}$ with its upper and lower bounds. Asymptotically, it has been shown that either the average hull dimensions $\mathtt{E}_{\mathscr{R}}(n,-1)$ is zero or it grows the same rate as n.

Keywords:
negacyclic codes; hulls; average $q-$dimension; reciprocal polynomials; semi local rings.

AMS Mathematics Subject Classification 2020:
11T71, 94B15.

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Non-commuting Graph of AC-groups Related to an Automorphism

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Abstract

Assume that α is an automorphism of finite group G and define an α-commutator of two elements $x, y \in G$ as $[x,y]_\alpha = x^{-1}y^{-1}xy^\alpha$. If $Z^\alpha(G) = \{x \in G \;:\; [y,x]_\alpha = 1, \forall y \in G\}$ is the α-center subgroup of G, then we define an α-noncommuting graph, denoted by $\Gamma^\alpha(G)$, with vertex set $G \setminus Z^\alpha(G)$. Two distinct vertices x and y are joined by an edge if and only if $[x,y]_\alpha \neq 1$ or $[y,x]_\alpha \neq 1$. It is clear that if we consider α as the identity automorphism, then the α-noncommuting graph is the ordinary one. In this paper, we consider G as an *AC*-group and α as an inner automorphism and study the structure of the α-noncommuting graph of *AC*-groups. Indeed, we investigate $\Gamma^\alpha(G)$ whenever $G/Z(G) \cong Z_p \times Z_p$, for a prime number p or dihedral group D_{2m}. Furthermore, we compute the chromatic number for the α-noncommuting graph of the quasi-dihedral group QD_{2^n}. It is important to notice that in these cases, the α-noncommuting graph is completely different from the ordinary noncommuting graph.

Keywords:
AC-group; Dihedral group; Noncommuting graph; p-group.

AMS Mathematics Subject Classification 2020:
05C25-05E16

[1]Speaker.

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Strongly Monolithic Characters of Finite Groups

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Abstract

Character theory of finite groups has an important place in understanding the structure of groups. In the literature, there are many publications from past to present on the relationships between the structure of finite groups and their complex irreducible characters. For example, J.G. Thompson proved in [1] that if every nonlinear irreducible character degree of a group G is divisible by a fixed prime number p, then G has a normal p-complement. It is also well-known and proven by Ito that a fixed prime number p does not divide all irreducible character degrees of a solvable group G if and only if G has a normal abelian Sylow p-subgroup. Sometimes it is not necessary to study with the set of all irreducible characters of a finite group to determine certain relations. For instance, the literature contains some results using only monolithic characters. Let G be a finite group and let χ be an irreducible complex character of G. If $G/ker\chi$ has only one minimal normal subgroup then the irreducible character χ is called a monolithic character of G. Gagola and Lewis proved in [2] that a group G is nilpotent if and only if $\chi(1)^2$ divides $|G:\ker\chi|$ for all irreducible characters χ of G. Later, Lu, Qin and Liu generalized in [3] this theorem for monolithic characters. In this talk, we will talk about the strongly monolithic characters, which are a subset of the set of monolithic characters of finite groups. In addition to giving some relationships

[1]Speaker.

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between the structure of finite groups and their strongly monolithic character. The concept of strongly monolithic character was first introduced in [4]. This is a joint work with Temha Erkoç and J.M. Özkan. The work in the talk was supported by (TÜBİTAK). The project number is 119F295.

Keywords:
Finite groups; solvable groups; nilpotent groups; strongly monolithic characters.

AMS Mathematics Subject Classification 2020:
20C15-20C20

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On the Planar Property of an Ideal Based Weakly Zero Divisor Graph

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Abstract

Let R be a commutative ring and I be an ideal of R. Beck [1] defined the zero-divisor graph of R as the graph whose vertices are zero-divisors of R, where two vertices x and y are adjacent if and only if xy=o. Anderson and Livingston [2] modified Beck's definition by excluding zero from the set of vertices and presented fundamental results on the zero divisor graph. The notion of a zero-divisor graph is generalized in [3], where products which are zero are replaced with products which are in some ideal I of R. The planar property of this generalized graph, called the ideal-based zero-divisor graph,

[1]author, Speaker.
[2]author.
[3]author.

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is studied in [3] and [9], and that of zero-divisor graph in [6] and [7]. As in [4], the weakly zero-divisor graph of R is the graph whose vertices are non-zero zero divisors of R and two vertices x and y are adjacent if and only if there exist r in ann(x) and s in ann(y) such that rs=0. We define an ideal-based weakly zero-divisor graph which contains the ideal-based zero-divisor graph as a subgraph and is identical to the weakly zero-divisor graph when I=0. We will determine when the ideal-based weakly zero-divisor graph is planar by introducing restraints on the size of I and girth of weakly zero-divisor graph of the factor ring R/I.

Keywords:
zero-divisor graph; commutative ring; planar graph; girth; graph theory.

AMS Mathematics Subject Classification 2020:
05C25, 20F65, 05C10

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Loops as semidierect product

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Abstract

In this talk, we will study a loop structure which is the semidirect product of a loop and a group. Let N be a loop, H be a group and $\phi : K \longrightarrow Sym(N)_1$ be a group homomorphism, where $Sym(N)_1$ is the stabilizer of 1 in $Sym(N)$. I will discuss the conditions under which a loop L would be a C-loop, a left Bol loop, a right Bol loop, a flexible loop, a left alternative loop, a right alternative loop and a left nuclear square loop. In addition, I will state a criterion under for the isomorphism of two semidirect products of a loop and a group.

Keywords:
Loops; Group; Semidirect Product; Isomorphism of loops.

AMS Mathematics Subject Classification 2020:
20N05

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On the average order of a finite group

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Abstract

Let $o(G)$ be the average of the element orders of a finite group G. A research topic concerning this quantity is understanding the relation between $o(G)$ and $o(H)$, where H is a subgroup of G. Let $\mathscr{N}$ be the class of finite nilpotent groups and let $L(G)$ be the subgroup lattice of G. We show that the set $\{\frac{o(G)}{o(H)} \mid G \in \mathscr{N}, H \in L(G)\}$ is dense in $[0, \infty)$. Other density results are outlined throughout the talk.

Keywords:
element orders; p-groups; nilpotent groups; density of a set.

AMS Mathematics Subject Classification 2020:
20D15-20D60-40A05.

[1]Speaker.

On the Sylow subgroups of amalgamated free products of pro-Q groups

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Abstract

In this paper, we investigate some free constructions of pro-Q groups. After pointing out some properties of complement subgroups and pification of profinite groups, we prove that some free pro-Q product of pro-Q groups can be written as semidirect product of its p-ification and a closed normal subgroup containing the normal complements of the p-Sylow of the factors. We recall the construction of free pro-Q product of pro-Q groups with amalgamation. We prove that this construction, for some pro-Q groups, is isomorphic to the amalgamated free pro-p product of the p-Sylow subgroups of the factors. Here, p is a prime number, Q a property on a class C of finite groups closed under taking subgroups, quotients, finite direct sums and exact sequences.

Keywords: pro-Q groups, p-ification, free pro-Q product of pro-Q groups with amalgamation, complement subgroups, Sylow subgroups

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Designs invariant under the simple groups $PSp_4(q)$

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(Thekiso T. Seretlo and Amin Saeidi)

Abstract

In this talk, we construct some designs invariant under the simple groups $PSp_4(q)$ for some prime power q. To achieve this, we use Key-Moori Method 1 and 2, to construct some designs from the two maximal subgroups and conjugacy classes of the projective symplectic groups $PSp_4(q)$. In the course, note that these two maximal subgroups have the same indices inside our groups, that is, $[PSp_4(q) : M] = q^3 + q^2 + q + 1$.

Keywords:
Projective symplectic groups; Key-Moori Methods; maximal subgroups; conjugacy classes; designs.

AMS Mathematics Subject Classification 2020:
20B15-05B05

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A Note on Autoconjugate Graphs of Finite Groups

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Abstract

Assume that g and h are two arbitrary elements of a given group G. Then g and h are said to be autoconjugate if there exists some automorphism α of G such that $h = \alpha(g)$. This concept introduced in 2016 and the authors constructed some bounds for the probability that two random elements of G are autoconjugate. They denoted this probability by $P_a(G)$. In this paper, we study this probability for some classes of finite groups, similar to dihedral groups and cyclic groups. We also, introduce a graph, Γ_G^a, associated to autoconjugacy classes of a finite group with vertex set $G \setminus L(G)$, where $L(G) = \{g \in G \ : \ g^{-1}\alpha(g) = 1, \ \forall \alpha \in Aut(G)\}$. Two distinct vertices x and y are adjacent by an edge if and only if they are autoconjugate. We may construct some new bounds for this probability by using the properties of this graph and also compute eigenvalues of the adjacency matrix of Γ_G^a for some finite groups.

Keywords:
Autocenter; Autoconjugate; Autoconjugate graph; Cyclic group; Dihedral group.

AMS Mathematics Subject Classification 2020:
05C25-05E16

[1]Speaker.

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CONSTRUCTING FISCHER-CLIFFORD MATRICES OF A MAXIMAL SUBGROUP $(2 \times 2_+^{1+8}){:}(U_4(2){:}2)$ OF Fi_{22} FROM ITS QUOTIENT GROUP $2^9{:}(U_4(2){:}2)$

L.KAPATA*, A. L. PRINS AND L. N. NJUGUNA

Abstract. The sporadic simple group Fi_{22} has a class of maximal subgroups of structure $\overline{G} = (2 \times 2_+^{1+8}){:}(U_4(2){:}2)$. The Frattini subgroup $\Phi(P) = \mathbb{Z}_2$ of the 2-group $P = 2 \times 2_+^{1+8}$ is characteristic in P and hence $\Phi(P) \lhd \overline{G}$. Therefore, the quotient group $\overline{Q} = \frac{\overline{G}}{\Phi(P)} \cong 2^9{:}(U_4(2){:}2)$ exists. In this talk, the Fischer-Clifford matrices $M(g_i)$ of $\overline{G}$ are constructed from the corresponding Fischer-Clifford matrices $\widehat{M(g_i)}$ of $\overline{Q}$ and we refer to this method as the *lifting of Fischer-Clifford matrices.*

MSC(2019): Primary: 20C15; Secondary: 20C40.

Keywords: coset analysis, Fischer-Clifford matrices, split extension, inertia factor, character table.

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On the (p,q,r)-generations of the group $G_2(3)$

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Abstract

A finite group G is said to be (p,q,r)-generated if it is generated by elements x and y such that the order of x is p, the order of y is q and the order of xy is r. The numbers p,q and r are prime divisors of the order of the group. The study/the presentation attempt to answer the question of finding all the (p,q,r)-generations [1] in respect of the group $G_2(3)$. The group $G_2(3)$ is one of the untwisted exceptional groups of Lie Type and it has ten maximal subgroups and nine conjugacy classes of elements of prime order [3]. Since its order is $4245696 = 2^6 \cdot 3^6 \cdot 7 \cdot 13$, the triples (p,q,r) with the condition that $p \leq q \leq r$ are constructed from the set $\{2,3,7,13\}$. The group $G_2(3)$ is simple hence it is not $(2,2,r)$-generated and also it is not a Hurwitz group since it is not $(2,3,7)$-generated [2].

Keywords:
(p,q,r)-generation; maximal subgroup; conjugacy class; finite simple group.

AMS Mathematics Subject Classification 2020:
20xxx-20yyy

[1] Speaker.

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$Z_2Z_2[u]Z_2[u]$-ADDITIVE CODES

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Abstract

Our study focuses on mixed alphabet codes over three alphabets Z_2 and its extension $Z_2[u] = Z_2 + uZ_2$, $u^2 = 0$. We introduce new Gray maps to examine cyclic codes and derive properties of their generators. Additionally, we construct binary linear codes and observe useful properties of their generators. Lastly we present examples of cyclic codes and binary linear codes.

Keywords:
Linear code, Binary code, Cyclic code, Constacyclic code, Gray image.

AMS Mathematics Subject Classification 2020:
11T71, 94B05.

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A note on the big lattices of classes of R-modules closed under submodules, quotients, and coproducts.

ABSTRACT: Philani Rodney Majozi
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A big lattice, which is defined as a proper class with a partial order, forms a lattice structure that is not a set. Recent research has investigated several notable big lattices. The big lattice of open classes, examined in Fernandez-Alonso and Raggi [2], exhibits distributive complete lattice properties despite not being a set. The study of the big lattice of Serre classes is presented in Stenström [6]. Bican, Kepka, and Němec [1] conduct an extensive exploration of the big lattice of preradicals in the category of modules over a ring R. Further investigations of the big lattice of preradicals in R-mod are conducted in Kasch [3], Raggi, Ríos, Rincón, Fernández-Alonso, and Signoret [4], and Raggi, Signoret, and Signoret [5]. Additionally, the big lattices of module classes R-sext and R-qext, introduced and investigated by Garcia et al represent classes of left R-modules closed under isomorphisms, submodules, extensions, and quotients. The results obtained in the study of two big lattices, $SC - R$ and $HC - R$, which represent classes closed under submodules, coproducts, and homomorphic images, coproducts, respectively, will be explored in this talk.

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On the breath of Lie superalgebras

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(Peyman Niroomand and Manuel Ladra)

Abstract

In this paper, we define the superbreath $b(L)$ of a finite-dimensional Lie superalgebra L and classify the structures of finite-dimensional nilpotent Lie superalgebras L with $b(L) \leq 1$.
Let L be a finite-dimensional Lie superalgebra. The superbreath $b(L)$ is equal to maximum of the dimension of the images of ad_x for all $x \in L$.
If a Lie superalgebra $L = L_{\bar{0}} \oplus L_{\bar{1}}$ is of dimension $m+n$, in which $\dim L_{\bar{0}} = m$ and $\dim L_{\bar{1}} = n$, then we write $\dim L = (m,n)$. Also, the superbreath $b(L) = b(L_{\bar{0}} + b(L_{\bar{1}})$ is equal to $r+s$ such that $b(L_{\bar{0}}) = r$ and $b(L_{\bar{1}}) = s$ if and only if $b(L) = (r,s)$.
First, we classify all nilpotent Lie superalgebras with $b(L) = (0,0)$ and we have L is an abelian Lie superalgebra if and only if $b(L) = (0,0)$.
The structure of finite-dimensional nilpotent Lie superalgebras L with $b(L) = (r,s)$ such that $r+s = 1$ are determined and proved $r+s = 1$ if and only if L is isomorphic to one of the following nilpotent Lie superalgebras

$$H_e = \langle x_1, \ldots, x_m, x_{m+1}, \ldots, x_{2m}, z \rangle \oplus \langle y_1, \ldots, y_n \rangle \oplus A(k)$$

[1]Speaker.

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with $[x_i, x_{i+m}] = z$ and $[y_j, y_j] = z$ for all i, j such that $1 \leq i \leq m, 1 \leq j \leq n$ or

$$H_o = \langle x_1, \ldots, x_m \rangle \oplus \langle y_1, \ldots, y_m, z \rangle \oplus A(k)$$

with $[x_i, y_i] = z$ for all i such that $1 \leq i \leq m$, where $A(k)$ is an abelian Lie superalgebra for $k \geq 0$.
Moreover, a relation between the superbreath of a Lie superalgebra L and its center are obtained. Let L be a finite-dimensional Lie superalgebra with $b(L) = (r, s)$ such that $r + s = n > 0$. Then there exists $x \in L$ such that $b(x) = n$ and

(i) if $[x, x] \neq 0$, then $\dim L/Z(L) \geq n$,

(ii) if $[x, x] = 0$, then $\dim L/Z(L) \geq n + 1$.

Keywords:
nilpotent Lie superalgebra; superbreath.

AMS Mathematics Subject Classification 2020:
17B30, 17B05, 17B99

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Groups having 12-cyclic subgroups

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Abstract

In the study of finite groups whenever there is a discussion about the subgroups, it is natural to consider their cyclic subgroups. A group G is said to be n-cyclic, if it contains n cyclic subgroups. It is easy to verify that G is 1-cyclic or 2-cyclic if and only if $G \cong \{e\}$, $G \cong \mathbb{Z}_p$, where p is a prime number respectively.
We looked at the additional effort done throughout time in counting these n-cyclic groups to further clarify this literature, beginning with the work by Tóth [4], which helped to count the number of cyclic subgroups of a finite abelian group. Later, Tărnăuceanu in [3] classified all finite groups G having $|G|-1$ number of cyclic subgroups. It is well known that G is $|G|$-cyclic if and only if G is an elementary abelian 2-group [1].
Zohu [5] found n-cyclic groups for $n = 3,4$ and 5. Later, Kalra [2] classified all the n-cyclic groups for $n = 6,7$ and 8 along with number of cyclic subgroups of groups of order p^k, pq and p^2q for $1 \leq k \leq 4$, where p and q are prime numbers. Recently, Ashrafi and Haghi [1] found all $9,10$-cyclic groups. In our earlier paper, we found all 11-cyclic groups. In the present work, all 12 cyclic groups are classified. A group G is 12-cyclic, if and only if $G \cong H$, where $H \in \{\mathbb{Z}_{p^{11}}, \mathbb{Z}_{p^5q}, \mathbb{Z}_{p^3q^2}, \mathbb{Z}_{p^2qr}, \mathbb{Z}_5 \times \mathbb{Z}_{25}, \mathbb{Z}_2 \times \mathbb{Z}_{32}, \mathbb{Z}_{25} \rtimes \mathbb{Z}_5, D_{16}, D_{18}, F_5, \mathbb{Z}_3.A_4, \mathbb{Z}_2 \times \mathbb{Z}_{2q^2}, \mathbb{Z}_{4q} \times \mathbb{Z}_2, Dic_6\}$ and p,q are prime numbers.

Keywords:
n-cyclic group; Cyclic subgroup; Sylow theorem; Maximal subgroup.

AMS Mathematics Subject Classification 2020:
20D20-20D25

[1]Khyati Sharma.

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Application of Hypergroup and H_v-group on Chemical Urea Formation Reactions

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Abstract

Frederic Marty in 1934 generalized the group structure to hypergroup. Since group structures have been generalized into hypergroups, generalization of algebraic structures called hyperalgebraic structures was born. Furthermore, hyperstructures are generalized again by Vougiouklis to H_v-structures by involving weak concepts in hyperstructures. Next, hyperstructures have applications, one of which is in chemistry to analyze types of hypergroups and H_v-groups in chemical reactions. Some of the chemical reactions that have been analyzed are dismutation reactions, redox reactions, ozone depletion reactions, and salt reactions. Inspired by this research, hypergroup and H_v-group types were investigated in the chemical reaction of urea formation. The results obtained in this study are the types of hypergroups and H_v-groups contained in the chemical reaction of urea formation are H_v-semigroup and semihypergroup.

Keywords:
Chemical Reactions; Hyperstructures; Hypergroup; Urea.

AMS Mathematics Subject Classification 2020: 20N20

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A classification of rational groups by using character degree graphs

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Abstract

A finite group G all of whose character values are rational is called a rational group. Equivalently, G is a rational group iff all generators of the cyclic group $\langle x \rangle$ are conjugate in G for every $x \in$ G. Thus, G is a rational group if and only if $N_G(\langle x \rangle)/C_G(\langle x \rangle) \cong \mathrm{Aut}(\langle x \rangle)$ for every $x \in$ G. For example, all symmetric groups, extra special 2-groups and elementary abelian 2-groups are rational groups. Manz, Willems and Wolf proved in [2] that the character degree graph of a finite group G has at most three connected components and if G is solvable, then the character degree graph has at most two connected components. In [3], Lewis has given a complete classification of solvable groups whose character degree graphs are disconnected. Then in [4], Lewis and White have discribed the structure of non-solvable groups whose character degree graphs are disconnected. Finally in [1], we determine all rational groups whose character degree graphs are disconnected.

Keywords:
Finite groups; rational groups, character degree graphs.

AMS Mathematics Subject Classification 2020:
20C15

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On a Group of the Form 2^{11}:M_{24}

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Abstract

The Conway Group Co_1 is one of the 26 sporadic simple groups. It is the largest of the three Conway groups with order $4157776806543360000 = 2^{21}.3^9.5^4.7^2.11.13.23$ and has 22 conjugacy classes of maximal subgroups. In this talk, we discuss a group of the form $\overline{G}$ =N:G, where $N = 2^{11}$ and $G = M_{24}$. This group $\overline{G}$ =N:G=2^{11}:M_{24} is a split extension of an elementary abelian group $N = 2^{11}$ by a Mathieu group $G = M_{24}$. Using the computed Fischer matrices for each class representative g of G and ordinary character tables of the inertia factor groups of G, we obtain the full character table of $\overline{G}$. The complete fusion of $\overline{G}$ into its mother group Co_1 is also determined using the permutation character of Co_1.

Keywords:
Conway group; Conjugacy classes; Fischer-Clifford matrices; Fusions; Permutation character.

AMS Mathematics Subject Classification 2020:
20C15, 20C34, 20D08.

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Central Automorphisms of Zappa-Szép Products

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Abstract

In this talk, I will discuss about the central automorphisms of the Zappa-Szép products of two groups. Also, I will give a description of the center of the Zappa-Szép product of two groups. As an application, I will compute the central automorphisms for a p-group of order p^5 which is the Zappa-Szép product of two groups of orders p^2 and p^3, where p is an odd prime.

Keywords:
Zappa-Szép product; Central Automorphisms; Automorphisms Groups; p=group.

AMS Mathematics Subject Classification 2020:
20D45

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On some codes and designs fromthe simple group U3(5)

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Abstract

We determine some codes and designs invariant under the Finite simple group U3(5) and attempt to give a geometric interpretation to words of minimum weight.

Keywords:
codes; designs; automorphism group; strongly regular graph; Stabilizer

AMS Mathematics Subject Classification 2020:
20xxx-20yyy

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Covering alternating groups by powers of cycle classes in symmetric groups

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Abstract

The study of products of conjugacy classes is a well established theme in the theory of finite groups. We consider symmetric groups and study when the subgroup of all even permutations in a symmetric group can be covered by powers of conjugacy classes which consist of cycles of fixed length. Given integers $k, l \geq 2$, where either l is odd or k is even, let $n(k,l)$ denote the largest integer n such that each element of A_n is a product of k many $l-$cycles. In [**?**], M. Herzog, G. Kaplan and A. Lev conjectured that $\lfloor \frac{2kl}{3} \rfloor \leq n(k,l) \leq \lfloor \frac{2kl}{3} \rfloor + 1$. It is known that the conjecture holds when $k = 2,3,4$ and some other partial results are also known. In [**?**] and [**?**], which are joint works with Harish Kishnani and Dr. Rijubrata Kundu, we completely determine the value of $n(k,l)$ in all the remaining cases. In particular, we show that the above conjecture does not hold in general.

Keywords:
alternating groups; symmetric groups; permutations; products of conjugacy classes.

AMS Mathematics Subject Classification 2020:
20B30, 220D06, 05A05, 220B05.

[1]Speaker.

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On the cyclic conjugacy class graph of groups

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Abstract

The cyclic conjugacy class graph of a group G is an undirected simple graph whose vertices are the nontrivial conjugacy classes of G such that two distinct vertices x^G and y^G are adjacent if and only if $\langle x', y' \rangle$ is cyclic for some $x' \in x^G$ and $y' \in y^G$. We discuss on the number of connected components as well as diameter of connected components of these graphs. Also, we consider the induced subgraph $\Delta_c(G)$ of the cyclic conjugacy class graph with vertices set $\{g^G | g \in G \setminus \mathrm{Cyc}(G)\}$, where $\mathrm{Cyc}(G) = \{g \in G : \langle x, g \rangle \mathit{cyclic\ for\ all}\ x \in G\}$, and classify all finite non-cyclic groups G with empty cyclic conjugacy class graphs.

Keywords:
cyclic; conjugacy class; graph; Triangle-free.

AMS Mathematics Subject Classification 2020:
Primary 05C25; Secondary 20E45.

1 Introduction

Herzog, Longobardi and Maj [4] defined the *commuting conjugacy class graph* (or CCC-graph) of a group G as a graph whose vertex set is the set of nontrivial

[1] Speaker.

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conjugacy classes of G such that two distinct vertices x^G and y^G are adjacent if $\langle x', y' \rangle$ is abelian for some $x' \in x^G$ and $y' \in y^G$. They have determined the connectivity of the CCC-graph of a group G and gave upper bounds for diameter of the corresponding connected components of the graph.

Likewise, we define the *cyclic conjugacy class graph* of G, denoted by $\Delta(G)$, as a simple undirected graph, in which the vertices are the nontrivial conjugacy classes of G and two distinct vertices x^G and y^G are adjacent whenever there exist two elements $x' \in x^G$ and $y' \in y^G$ such that $\langle x', y' \rangle$ is cyclic.

Let G be a group. For each $x \in G$, the *cyclizer* of x is defined as

$$Cyc_G(x) = \{y \in G : \langle x, y \rangle \text{ is cyclic}\}$$

In addition, the *cyclizer* of G is defined by

$$Cyc(G) = \bigcap_{x \in G} Cyc_G(x)$$

Cyclizers were introduced by Patrick and Wepsic in [3] and studied in [1, 2]. It is known that $Cyc(G)$ is a cyclic normal subgroup of G and $Cyc(G) \subseteq Z(G)$.

2 Main Results

Lemma 2.1. *Let G be a finite group and $x, y \in G \setminus \{1\}$ such that $xy = yx$ and $(|x|, |y|) = 1$. Then $x^G \leftrightarrow y^G$.*

Theorem 2.2. *Let G be a finite group such that $Z(G)$ is not a p-group. Then $\Delta(G)$ is connected. Moreover,* $\operatorname{diam}(\Delta(G)) \leq 4$ *and the bound is sharp.*

Theorem 2.3. *Let G be a finite p-group. Then there exists a one-to-one correspondence between the connected components of $\Delta(G)$ and the minimal cyclic subgroups of G which are not in the same conjugacy class.*

Theorem 2.4. *Let G be a finite nilpotent group.*

(1) If G is a p-group, then the number of connected components of $\Delta(G)$ is the same as the number of subgroups of G of order p which are not in the same conjugacy class. In particular, $\Delta(G)$ is connected if and only if G is a cyclic p-group or a generalized quaternion 2-group,

(2) If G is not a p-group and each of the Sylow p-subgroups of G is a cyclic p-group or a generalized quaternion 2-group, then $\Delta(G)$ is connected and $\operatorname{diam}(\Delta(G)) = 2$.

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(3) *If G is not a p-group and G has a Sylow p-subgroup, which is neither a cyclic p-group nor a generalized quaternion 2-group, then $\Delta(G)$ is connected and* $\mathrm{diam}(\Delta(G)) \leq 3$.

In 2009 Herzog et al. considered the induced subgraph $\Gamma_c(G)$ of CCC-graph with vertices set $\{g^G | g \in G \setminus Z(G)\}$ and proved that if G is a periodic non-abelian group, then $\Gamma_c(G)$ is an empty graph if and only if G is isomorphic to one of the following groups S_3, D_8 or Q_8. Recently, Mohammadian and et al. [5] classified all finite non-abelian groups G with triangle-free CCC-graphs. Now, we consider the induced subgraph $\Delta_c(G)$ of $\Delta(G)$ with vertices set $V(\Delta(G)) \setminus \{g^G | g \in \mathrm{Cyc}(G)\}$ and classify all finite non-cyclic groups G with empty cyclic conjugacy class graphs.

Lemma 2.5. *Let G be a periodic non-cyclic group such that $\Delta_c(G)$ is an empty graph. Then every nontrivial element of $\frac{G}{Cyc(G)}$ is of prime order. Moreover, either $Cyc(G) = 1$ or G is 2-group.*

Theorem 2.6. *Let G be a periodic non-cyclic group and $\Delta_c(G)$ is an empty graph. Then G is isomorphic to one of the following groups*

(i) Elementary abelian 2-group,

(ii) $\mathbb{Z}_3^m \rtimes \mathbb{Z}_2$,

(iii) Q_8.

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A kind of g-noncommuting graph

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Abstract

Let G be a finite group. We define a graph associated to a fixed element g of G and denote by $\Delta_g(G)$. The vertex set is all of non-central elements and two distinct vertices x and y join with an edge if and only if $[x,y] \notin \langle g \rangle$, which $[x,y] = x^{-1}y^{-1}xy$. In this talk, we state some graph theoretical properties on this graph such as diameter, clique and independence numbers.

Keywords:
Commutator; Dihedral group; Connected graph.

AMS Mathematics Subject Classification 2020:
20P05

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Lattice matrices and linear transformation of lattice vector spaces

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Abstract

In this paper, we demonstrate how a lattice vector space is formed from the set of all ideals of a distributive lattice. We also show that the set of all join/meet (or both) homomorphisms form a lattice vector space. Later we introduce the idea of congruence relation. Using this we define quotient spaces. We prove results that highlight the connection between lattice matrices and linear transformations in lattice vector spaces.

Keywords:
Distributive lattice, lattice vector spaces, lattice matrices.

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On superfluous ideals of N-groups and related graph

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Abstract

Let N be a right nearring N and G be an N-group. For an arbitrary ideal (or N-subgroup) Ω of an N-group G, we define the notions Ω-superfluous, strictly Ω-superfluous, g-superfluous ideals of G. We provide appropriate examples to distinguish among these classes and the existing classes. We obtain one-one correspondence between the superfluous ideals of an N-group (over itself) and those of $M_n(N)$-group N_n, where $M_n(N)$ is the matrix nearring over N. Furthermore, we define a graph of superfluous ideals of a nearring and prove some properties with necessary examples.

Keywords:
Nearring;N-group; superfluous ideals; matrix nearring.

AMS Mathematics Subject Classification 2020:
16Y30

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Generalised essential ideal graph of an N-group

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Abstract

We consider a module over right nearring[3], denoted by H and define the notion of g-essential ideal of H, which is a generalization of essential ideal[4]. We also define a graph based on g-essential ideal by taking the set of all non-trivial ideals as vertices and an edge is defined between two distinct vertices whenever the sum is g-essential (We denote such a graph by $\mathscr{L}_g(H)$). We discuss the combinatorial properties such as connectivity, diameter, completeness of $\mathscr{L}_g(H)$. We prove a characterization for $\mathscr{L}_g(H)$ to be complete. We also prove $\mathscr{L}_g(H)$ has diameter at-most 2 and obtain related properties with suitable illustrations. Further, we consider the subgraph of $\mathscr{L}_g(H)$ which is induced by the set of all non-g-essential ideals and prove that there exists a path between two superfluous ideals.

Keywords:
nearring; superfluous; essential; graph; ideals.

AMS Mathematics Subject Classification 2020:
16Y30, 05C25

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On N-groups with essential ideals and superfluous ideals

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Abstract

We consider an N-group H where N is a zero-symmetric right nearring. We define the notions of essential ideal, superfluous ideal, generalized essential ideal of a H. We discuss the combinatorial properties such as connectivity, diameter, completeness of a graphs based on these ideals of N-group. We prove a characterization for a generalized essential ideal graph to complete. We also prove that this graph has diameter at-most 2 and obtain related properties with suitable illustrations. We also prove some related results on matrix maps over N.

Keywords:
Nearring;N-group; superfluous ideals; matrix nearring.

AMS Mathematics Subject Classification 2020:
16Y30

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Chinese Remainder Theorem for Hypernearrings

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Abstract

The additive substructure of a nearring is a group whereas for a hypernearring it is a quasi canonical hypergroup. We prove the Chinese remainder theorem for nearrings and hypernearrings. We give examples of nearrings and hypernearrings of order 2^n; which exhibit fractal patterns with respect to multiplication, and then illustrate the Chinese remainder theorem for these newly found examples.

Keywords:
Chinese remainder theorem; Nearring; Hypergroup; Hypernearring; Ideal; Fractal

AMS Mathematics Subject Classification 2020:
16Y30, 16Y20

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Undeniable Signature Scheme Based on DLFP Over Semiring

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Abstract

D. Chaum and H. Van Antwerpen proposed the Undeniable signature scheme in 1989. During the verification process, the Co-operation of the signatory is necessary which serves as the scheme's speciality. In this paper, novel undeniable signature scheme based on DLFP over semiring is proposed. Also, the security and complexity analysis of the scheme is provided.

Keywords:
Undeniable signature scheme; Key exchange protocol; Diffie-Hellman; Semiring; Factor problem.

AMS Mathematics Subject Classification 2020:
11T71, 11Z05, 12E20

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A Public Key Cryptosystem and a Key Exchange Protocol Based on Invertible Matrices over Semiring

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Abstract

Diffie-Hellman unfolded the Key distribution problem with a quick fix where the entities exchange shared secret key without any prior communication through an unreliable channel. In this paper, a key exchange protocol based on Factorization Discrete Logarithm Conjugacy Search Problem (FDLCSP) over semiring is proposed. The security and complexity analysis of the proposed protocol is examined. Also, based on the proposed key exchange protocol, an ElGamal cryptosystem is presented.

Keywords:
Public key cryptography; Key exchange protocol; Diffie-Hellman; ElGamal cryptosystem; Semiring.

AMS Mathematics Subject Classification 2020:
11T71, 11Z05, 12E20

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