Operator Theory

and

Matrix Analysis Workshop

In honor of Gilbert Groenwald's 65th birthday

17-18 November 2020, online

Program

Time	**Tuesday 17 November**
13:45–14:00	opening
14:00–14:45	Rien Kaashoek
	break
15:00–15:30	Jacob Jaftha
15:30–16:00	Christian Budde
16:00–16:30	Manfred Möller
	break
17:00–17.30	Alma Naude
17:30–18:00	Madelein van Straaten
	break
18:15–19:00	Hugo Woerdeman
19:00–19:15	laudatio

Time	**Wednesday 18 November**
13:45–14:15	Louis Labuschagne
14:15–14:45	Miek Messerschmidt
	break
15:00–15:30	Elroy Zeekoei
15:30–16:00	Eder Kikianty
16:00–16:30	Bruce Watson
	break
17:00–17.30	Sanne ter Horst
17:30–18:00	Dawie Janse van Rensburg
18:00–18:30	André Ran
	break
18:45–19:30	Joe Ball
19:30–19:45	closing

Times are for South African time zone (GMT+2)

Abstracts Tuesday 17 November

Rien Kaashoek *Operator reverse problems inspired by the theory of Toeplitz matrices*

In 1972 Israel Gohberg and A.A. Semenčul solved a beautiful reverse problem, starting from one column vector x in $\mathbb{C}^n$ and involving self-adjoint $n \times n$ Toeplitz matrices T as unknowns. The corresponding equation is $Tx = c^*$, where c is the one row vector $1, 0, \ldots, 0$. In the present talk the role of Toeplitz matrices is taken over by contraction-structured operators, using the theory of contractions and corresponding dilation theory. In fact, if A is a contraction on a Hilbert space $\mathcal{X}$, then an operator T on $\mathcal{X}$ is called a *self-adjoint A-structured operator* if T is self-adjoint and

$$P_{\mathrm{Ker}\, C}(T - ATA^*)P_{\mathrm{Ker}\, C} = 0, \text{where } I - AA^* = C^*C.$$

The corresponding equation is $TX = C^*$, where $C : \mathcal{X} \to \mathcal{E}$ is surjective and $X : \mathcal{E} \to \mathcal{X}$ is given. Solutions will be given for two different classes of contractions A. The talk is based on joint work with Art Frazho and Freek van Schagen.

Jacob Jaftha *My journey with Gilbert: from vector spaces to Toeplitz-like operators*

Starting with a few personal anecdotes of my interaction with Gilbert Groenewald on my journey in maths leading to our joint work in Toeplitz-like operators. We will discuss Fredholm properties of a Toeplitz-like operator with rational matrix symbol having poles on the unit circle based on a Wiener-Hopf factorization of the symbol. Some differences with the bounded Toeplitz operator with rational matrix symbol will be highlighted. This is joint work with Gilbert, Sanne ter Horst and Andre Ran.

Christian Budde *Extrapolation of operator-valued multiplication operators*

We provide some motivation for studying operator-valued multiplication operators on Bochner L^p-spaces as well as their extrapolation. In fact, we will combine these two notions using so-called Banach fiber spaces. We will show, that the extrapolation does occur along fibers. This is joint work with R. Heymann (Stellenbosch/South Africa).

Manfred Möller *Minimality of eigenfunctions and associated functions of ordinary differential operators.*

Let E, F and H be infinite-dimensional Banach spaces such that E is a dense subset of H with continuous embedding $E \hookrightarrow H$. The dual spaces are denoted by E', F', H', and the corresponding bilinear forms will be denoted by $\langle\ ,\ \rangle$. A sequence $(w_i)_{i=1}^n$ in H is called minimal in H if $w_j \notin \overline{\operatorname{span}\{w_i : i \in \mathbb{N},\, i \neq j\}}$ for all $j \in \mathbb{N}$.

Let $B \in L(E, F)$. Two sequences $(y_i)_{i=1}^\infty$ in E and $(v_i)_{i=1}^\infty$ in F' are called B-biorthogonal if

$$\langle By_i, v_j \rangle = \delta_{ij} \quad (i, j \in \mathbb{N}).$$

Let $F = F_1 \times F_2$, with a Banach space F_1 and and finite dimensional space F_2 of dimension k, where $k = 0$ is allowed. Then we can write

$$B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

with $B_l \in L(E, F_l)$ $(l = 1, 2)$.

Further it is assumed that B_1 has a continuous extension $B_0 \in L(H, F_1)$. With respect to the decomposition $F = F_1 \times F_2$ we write $v_i = (v_{i,1}, v_{i,2})$ $(i \in \mathbb{N})$.

Theorem Assume that $\operatorname{span}\{v_{1,2}, \ldots, v_{k,2}\} = F_2$. Then $(y_i)_{i=k+1}^\infty$ is minimal in H.

This abstract result can be applied to regular ordinary linear differential operators in Sobolev spaces whose boundary conditions may depend linearly on the eigenvalue parameter for proving minimality in L_p.

Explicit examples for Sturm-Liouville problems with one or both boundary conditions depending on eigenvalue parameter are considered.

These results have been published in Advances in Operator Theory, 5 (2020), 1014 - 1025.

Alma Naude *Hill representations and positive linear matrix maps that are completely positive.*

Let $\mathbb{F} = \mathbb{C}$ or $\mathbb{F} = \mathbb{R}$. In [1], [2], R.D. Hill studied linear matrix maps $\mathcal{L} : \mathbb{F}^{n\times n} \to \mathbb{F}^{q\times q}$ of the form

$$\mathcal{L}(V) = \sum_{k,l=1}^{m} \mathbb{H}_{kl}\, A_k V A_l^*, \quad V \in \mathbb{F}^{n\times n}, \tag{1}$$

with $A_1, \ldots, A_m \in \mathbb{F}^{q\times n}$ linearly independent. We refer to (1) as a *minimal Hill representation* of $\mathcal{L}$ and $\mathbb{H} = [\mathbb{H}_{kl}]_{k,l=1}^m \in \mathbb{F}^{m\times m}$ the associated *Hill matrix.* We say a linear map $\mathcal{L}$ is $*$-linear if it respects adjoints $(\mathcal{L}(V)^* = \mathcal{L}(V^*))$. By Theorem 4 (resp. Theorem 3) in [3] we know a linear map $\mathcal{L}$ as in (1) is $*$-linear (resp. completely positive) if and only if $\mathbb{H}$ is Hermitian (resp. positive semidefinite).

We provide a further analysis of minimal Hill representations for $*$-linear maps. In particular, we pay attention to how minimal Hill representations of a $*$-linear map $\mathcal{L}$ can be constructed and connect them to the matricization L and Choi matrix $\mathbb{L}$ of $\mathcal{L}$. Furthermore, we make use of structural properties needed on L for $\mathcal{L}$ to be a $*$-linear map to obtain special cases for which positivity and complete positivity of $\mathcal{L}$ coincide.

[1] R.D. Hill, Inertia theory for simultaneously triangulable complex matrices, *Linear Algebra Appl.* **2** (1969), 131–142.

[2] R.D. Hill, Linear transformations which preserve hermitian matrices, *Linear Algebra Appl.* **6** (1973), 257–262.

[3] J.A. Poluikis and R.D. Hill, Completely positive and Hermitian-preserving linear transformations, *Linear Algebra Appl.* **35** (1981), 1–10.

Madelein van Straaten *mth roots of H-selfadjoint quaternion matrices*

A square matrix B is called H-selfadjoint for some invertible Hermitian matrix H if it is selfadjoint in the corresponding indefinite inner product space, or equivalently, if $HB = B^*H$. Let B be an H-selfadjoint quaternion matrix. It is well known that there exists a complex matrix representation for quaternion matrices and this simplifies the process of finding necessary and sufficient conditions for the existence of an H-selfadjoint quaternion matrix A such that $A^m = B$, that is, A is an mth root of B.This process is illustrated in the talk and the conditions for the existence of an H-selfadjoint mth root are given.

Hugo Woerdeman *Error Bounds and Singularity Degree in Semidefinite Programming.*

For certain pathological instances of semidefinite programming, state-of-the-art algorithms, while theoretically guaranteed to converge to a solution, do so very slowly or can fail to converge entirely. This issue is exacerbated in that it is generally undetectable. In this paper we propose a method to detect this type of slow convergence by lower bounding forward error, i.e., distance to the solution set. This bound is obtained by analyzing a class of parametric curves that are proven to converge to a solution of maximum rank and then upper bounding that rank. This talk is based on joint work with Stefan Sremac and Henry Wolkowicz.

Abstracts Wednesday 18 November

Louis Labuschagne *Fredholm properties of Toeplitz maps on noncommutative H^2 spaces*

We describe the construction of H^p spaces for group von Neumann algebras before going on to investigate the plausibility of recovering the famous Gohberg-Krein index theorem for Fredholm Toeplitz maps in this context. Whilst much of the classical theory does go through, there are also strong indications that there exist group algebras which admit no Fredholm Toeplitz maps. In closing we present existence criteria for compact Hankel maps, also indicating how such existence ensures the existence of Fredholm Toeplitz maps.

Miek Messerschmidt *On compact packings of the plane by circles*

Consider the following problem which is currently open for all values $n \geq 4$:

Let $n \in \mathbb{N}$. How many configurations of distinct numbers

$$0 < r_0 < r_1 < \ldots < r_{n-2} < r_{n-1} = 1$$

admit a compact packing of the plane by circles with radii chosen from the set $\{r_0, r_1, \ldots, r_{n-1}\}$ and with at least one circle of each radius r_i occurring in the packing?

We will discuss: (0) The definition of a compact packing of the plane by circles. (1) Some of this problem's history. (2) The tools involved in solving the problem in the cases for $n = 1, 2, 3$. (3) Some further problems that arise in attacking the problem in the general case, which includes some problems in matrix analysis.

Elroy Zeekoei *Summability of sequences and extensions of operator ideals*

Given a Banach space Y, the so-called right$Y$ and left Y–extensions of operator ideals are introduced and applied to the study of operator ideal properties of well-known classes of operators on Banach spaces.

The key is to introduce a general theory (in the context of operator ideal theory) to study some classes of operators which are right or left Y-extensions of classical operator ideals for some Banach space Y. This leads to a study of a class of operators, called $(r; Yq)$- limited operators, which generalizes some recently introduced operators in the literature.

In particular, the results of this study are applied to obtain characterizations and new information about the sequentially p-limited operators, weakly mid $(q; p; r)$-summing operators, p-convergent (or p-converging) operators and weak* p-convergent operators.

Eder Kikianty *Some constants for Banach spaces*

For any $1 < p < \infty$, the L^p-spaces are uniformly convex, but L^2 is "more so" than others. The notion of uniform convexity does not capture this difference. The modulus of convexity, however, provides us with more "quantitative" information on the convexity. In this talk, I will provide an overview on some constants for Banach spaces and their relationship with various properties of the given space. In particular, I will discuss the James constant, the von Neumann-Jordan constant, and the Dunkl-Williams constant.

Bruce Watson *The indefinite p-Laplacian.*

In this talk we will look at the point spectrum of the 1-dimensional p-Laplacian on a compact interval and the extension of the results to the half line. In addition to this oscillation theory and asymptotics will be discussed.

Sanne ter Horst *Equivalence after extension and Schur coupling for relatively regular operators*

It was recently shown that the relations on Banach space operators know as Schur Coupling (SC) and Equivalence After Extension (EAE) do not coincide in general. The counter example followed from an analysis of Fredholm operators on essentially incomparable Banach spaces, for instance the forward shift operator on different ℓ^p spaces. More generally one can consider the class of relatively regular operators, also known as generalised invertible operators. For this class of operators we present an equivalent Banach space operator problem and provide some analysis of this problem, leading to some new cases where EAE and SC coincide, and also one where they do not.

The talk is based on joint work with M. Messerschmidt and A.C.M. Ran.

Dawie Janse van Rensburg *An alternative canonical form for quaternionic H-unitary matrices*

The field of linear algebra over the quaternions is a research area which is still in development. In this paper we continue our research on canonical forms for a matrix pair (A, H), where the matrix A is H-unitary, H is invertible and with A as well as H quaternionic matrices. We seek an invertible matrix S such that the transformations from (A, H) to $(S^{-1}AS, S^*HS)$ brings the matrix A in Jordan form and simultaneously brings H into a canonical form. Canonical forms for such pairs of matrices already exist in the literature, the goal of the present paper is to add one more canonical form which specifically keeps A in Jordan form, in contrast to the existing canonical forms.

André Ran *Wiener-Hopf factorization indices of unitary rational matrix functions on the unit circle*

In this talk rational matrix functions that have unitary values on the unit circle are discussed. The factorization indices of such functions will be described in terms of the matrices appearing in realizations of the factors in a special factorization of the function. The notion of matricial coupling plays a central role in this analysis. The talk is based on joint work with Rien Kaashoek and Gilbert Groenewald.

Joe Ball *Free Noncommutative Hereditary Kernels: Jordan Decomposition and Kernel Positivstellensätze (nc Nevanlinna-Pick interpolation in disguise)*

We discuss a quantized version of the Jordan decomposition theorem for a complex Borel measure on a compact Hausdorff space, namely, the more general problem of decomposing a general noncommutative kernel (a quantization of the standard notion of kernel function) as a linear combination of completely positive noncommutative kernels (a quantization of the standard notion of positive definite kernel). Other special cases include: the problem of decomposing a general operator-valued kernel function as a linear combination of positive kernels (not always possible), of decomposing a general bounded linear Hilbert-space operator as a linear combination of positive linear operators (always possible), of decomposing a completely bounded linear map from a C^*-algebra $\mathcal{A}$ to an injective C^*-algebra $\mathcal{B}$ as a linear combination of completely positive maps from $\mathcal{A}$ to $\mathcal{B}$ (always possible). We also discuss a noncommutative kernel version of a Positivstellensatz (i.e., finding a certificate to explain why one kernel is positive at points where another given kernel is positive), and show how the recent solution of the Nevanlinna-Pick interpolation problem in the setting of free noncommutative functions can be viewed as a special case of such a kernel Positivstellensatz. This is joint work with Gregory Marx and Victor Vinnikov.